## Andromeda \& Milky Way galaxies in free fall towards each other IF initial mutual velocity $=0 \mathrm{~m} / \mathrm{s}$ (both radial and orbital velocity).

 (Andromeda actually has a radial velocity towards us of rougly $300 \mathrm{~km} / \mathrm{s}$, but we hardly know its lateral velocity, so it might well be that we are revolving our common barycentre).We consider the resulting bull's eye trajectory to be a degenerate elliptical orbit $[a<\infty, b=0 \Rightarrow e=1]$ from apoapsis to periapsis according to Kepler's laws, where the reduced mass $\mu=M m /(M+m)$ "orbits" the $M+m$ barycentre and we calculate half the orbital period.

$$
\begin{gathered}
\text { Kepler's } 3^{\text {rd }} \text { law: } \omega^{2} a^{3}=G M_{\mathrm{tot}} \therefore \omega=\sqrt{\frac{G M_{\mathrm{tot}}}{a^{3}}} \\
\text { we have: } a=\frac{R}{2} \therefore a^{3}=\frac{R^{3}}{2^{2} 2} \\
\omega=\frac{2 \pi}{T} \therefore T=\frac{2 \pi}{\omega}=2 \pi \cdot \sqrt{\frac{a^{3}}{G M_{\mathrm{tot}}}}=\frac{2 \pi}{2} \cdot \sqrt{\frac{R^{3}}{2 G M_{\mathrm{tot}}}}
\end{gathered}
$$

hence, the free fall time ( $1 / 20 \mathrm{orbit}$ ) is: $\quad \boldsymbol{t}_{\mathrm{ff}}=\frac{T}{2}=\frac{\pi}{2} \cdot \sqrt{\frac{R^{3}}{2 G M_{\mathrm{tot}}}}$
We have:

$$
\begin{gathered}
R \approx 2.54 \times 10^{6} \mathrm{ly} \approx 2.403 \times 10^{22} \mathrm{~m} \\
G \approx 6.67430 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2} \\
M_{\mathrm{MW}} \approx 1.15 \times 10^{12} M_{\odot} \\
M_{\text {Andr }} \approx 1.5 \times 10^{12} M_{\odot} \\
M_{\odot} \approx 1.98847 \times 10^{30} \mathrm{~kg} \\
M_{\text {tot }}=M_{\mathrm{MW}}+M_{\text {Andr }} \approx 5.27 \times 10^{42} \mathrm{~kg}
\end{gathered}
$$

yielding:

$$
t_{\mathrm{ff}} \approx 2.21 \times 10^{17} \mathrm{~s} \approx 6.99 \times 10^{9} \mathrm{yr} \approx 0.508 t_{\mathrm{H}}
$$

At the aforementioned speed of $300 \mathrm{~km} / \mathrm{s}$ (wrongly assuming it constant), it would take $2.403 \times 10^{22} \mathrm{~m} / 300 \mathrm{~km} / \mathrm{s} \approx 2.54 \times 10^{9} \mathrm{yr} \approx 0.184 \mathrm{t}_{\mathrm{H}}$.

Andromeda is our nearest neighbour spiral galaxy. On the cosmic scale, it is very close to us and yet our kiss (starting with zero velocity) would have to bide

## seven billion years, half the age of the universe!

## Intergalactic gravitation is <br> gIGANTICALLY <br> Small.

Yes, it's astronomically small, negligible for most purposes.

## We see this fleet need no beat and neatly meet \& the speed be:

(of course it's a one-dimensional head-on collision and we calculate in the barycentric frame!)
conservation of energy: $\quad E_{\text {kin,tot }}=\frac{1}{2} M_{\mathrm{A}} v_{\mathrm{A}}^{2}+\frac{1}{2} M_{\mathrm{MW}} v_{\mathrm{MW}}^{2}=\frac{G M_{\mathrm{A}} M_{\mathrm{MW}}}{R}=E_{\text {pot }}$
conservation of momentum:

$$
\frac{v_{\mathrm{A}}}{v_{\mathrm{MW}}}=\frac{M_{\mathrm{MW}}}{M_{\mathrm{A}}}
$$

(using absolute values)
hence:
therefore:
$v_{\mathrm{MW}}=v_{\mathrm{A}} \frac{M_{\mathrm{A}}}{M_{\mathrm{MW}}} \therefore v_{\mathrm{MW}}^{2}=v_{\mathrm{A}}^{2} \frac{M_{\mathrm{A}}^{2}}{M_{\mathrm{MW}}^{2}}$
$\frac{1}{2} M_{\mathrm{A}} v_{\mathrm{A}}^{2}+\frac{1}{2} M_{\mathrm{MW}} v_{\mathrm{A}}^{2} \frac{M_{\mathrm{A}}^{2}}{M_{\mathrm{MW}}^{2}}=\frac{G M_{\mathrm{A}} M_{\mathrm{MW}}}{R}$
i.e.:
$v_{\mathrm{A}}^{2} M_{\mathrm{A}}\left(1+\frac{M_{\mathrm{A}}}{M_{\mathrm{MW}}}\right)=\frac{2 G M_{\mathrm{A}} M_{\mathrm{MW}}}{R}$
yielding:
and:
$v_{\mathrm{A}}=\sqrt{\frac{2 G M_{\mathrm{A}} M_{\mathrm{MW}}}{R M_{\mathrm{A}}\left(1+\frac{M_{\mathrm{A}}}{M_{\mathrm{MW}}}\right)}}=\sqrt{\frac{2 G M_{\mathrm{MW}}^{2}}{R\left(M_{\mathrm{A}}+M_{\mathrm{MW}}\right)}}=M_{\mathrm{MW}} \sqrt{\frac{2 G}{R\left(M_{\mathrm{A}}+M_{\mathrm{MW}}\right)}}$
which results in:
This renders:
$v_{\mathrm{MW}}=v_{\mathrm{A}} \frac{M_{\mathrm{A}}}{M_{\mathrm{MW}}} \quad=M_{\mathrm{A}} \sqrt{\frac{2 G}{R\left(M_{\mathrm{A}}+M_{\mathrm{MW}}\right)}}$

Cf.:
$\boldsymbol{v}_{\text {coll }}=\left(M_{\mathrm{A}}+M_{\mathrm{MW}}\right) \sqrt{\frac{2 G}{R\left(M_{\mathrm{A}}+M_{\mathrm{MW}}\right)}}=\sqrt{\frac{\mathbf{2 \boldsymbol { G } ( \boldsymbol { M } _ { \mathbf { A } } + \boldsymbol { M } _ { \mathbf { M W } } )}}{\boldsymbol{R}}}$
~171 km/s
$\sim 200 \mathrm{~km} / \mathrm{s}<v_{\odot, \mathrm{MW}}<\sim 250 \mathrm{~km} / \mathrm{s}$
Dividing the sum of their radii by this velocity yields the maximum duration of this collision:

| we find: | $R_{\mathrm{MW}}$ | $\approx 52850 \mathrm{ly}$ | (Google as of 2024-03-26) |
| :--- | :--- | :--- | :--- |
| and: | $R_{\mathrm{A}}$ | $\approx 110000 \mathrm{ly}$ | (sic) |
| so: | $R_{\mathrm{tot}}$ | $\approx 162850 \mathrm{ly} \approx 1.54 \times 10^{21} \mathrm{~m}$ |  |

yielding:
$\Delta t_{\text {coll }}<\sim 285 \underline{\text { million years. }}$
How many stars will collide? See http://henk-reints.n//astro/HR-Galaxy-star-collision.pdf.
Now assume the sun's orbital period around the galactic centre ( $\sim 240 \mathrm{Ma}$ ) equals the rotation period of the entire Milky Way. Then the latter has revolved not more than a mere $13.77 \mathrm{Ga} / 240 \mathrm{Ma} \approx 57$ times since the big bang.


Ceci n'est pas la galaxie d'Andromède, ni la voie lactée.

