Classical thermodynamics yields: $E_{class} = \frac{3}{2}kT_{class} = \frac{1}{2}mv^2 = mc^2\frac{\beta^2}{2}$

Relativistic *kinetic energy*: $E_{rel} = \gamma m_0c^2 - m_0c^2 = m_0c^2(\gamma - 1) = m_0c^2\left(\frac{\beta^2}{2} + \mathcal{O}(\beta^4)\right)$

hence: $\gamma = 1 + \frac{E_{rel}}{m_0c^2}$

yielding what I'll call a *relativistic temperature* of:

$$T_{rel} = \frac{2E}{3k} = \frac{2m_0c^2(\gamma-1)}{3k}$$

where γ of course corresponds to the *thermal velocity* of the molecules.

This so found *relativistic temperature* would – just like in classical physics – be able to reach infinity, but I think physical limitlessness is fundamentally impossible, so I ~~define~~ pick from thin air what I will call the

relativistic Boltzmann constant: $\boldsymbol{k_{rel} \coloneqq \gamma k}$

just in order to investigate what it could mean.

It renders: $T_{rel} = \frac{2m_0c^2(\gamma-1)}{3k_{rel}} = \frac{2m_0c^2}{3k} \cdot \frac{(\gamma-1)}{\gamma}$

yielding: $T_{max} = \lim_{\gamma\to\infty} T_{rel} = \frac{2m_0c^2}{3k}$

This would be the *temperature* of an ideal gas where each molecule would have a *thermal energy* equal to its own relativistic *mass equivalent* and it would for any given m_0 be a true upper *temperature* limit.

For $m_0 = 1$ Da we obtain: $T_{max,1u} = \frac{2}{3k} \cdot (1\ \text{Da}) \cdot c^2 \approx 7.206 \times 10^{12}$ K.

This value of $m_0 = 1$ Da roughly applies to monatomic hydrogen. The lightest hadron is π^0 with a *mass* of $134.8766\ \text{MeV}/c^2 \approx 0.14479598$ Da, yielding: $T_{max,\pi^0} = \frac{2}{3k} \cdot 134.8766\ \text{MeV} \approx 1.043 \times 10^{12}$ K. I think that, at such a *temperature*, molecules more massive than nucleons cannot exist for a reasonable amount of *time*. They will either be blown apart by the collisions or decay very rapidly. Practically all known particles are unstable with *lifetimes* of: neutron: 15 min which I'll call quasi stable, muon: 2.2 µs (in which light travels just 660 m), K-Long: 52 ns (≈ 15.6 m light travel distance), and all others have a (far) shorter *lifetime*, which means they can hardly be called "existing".

These T_{max} values are all similiar to the *Hagedorn temperature*[1] (a sort of "melting point" where spontaneous pair production occurs): $T_H = 158$ MeV or 1.222×10^{12} K. WikipediA[2] gives 150 MeV and 1.7×10^{12} K (which cannot be correct since $150\ \text{MeV} \times \frac{2}{3k} = 1.16 \times 10^{12}$ K).

Can it still be called a rise in *temperature* if addition of *energy* no longer results in more movement of the particles but in more moving particles? The term *temperature* applies only to stochastical movement of molecules relative to one another (or to their common "local barycentre"). I do not consider it a *temperature* if molecules are in ballistic motion, i.e. travelling together at a high speed w.r.t. some macroscopic reference point, but with small relative motion w.r.t. each other. For example the 7 TeV that the LHC pumps into each proton would correspond to 5.4×10^{16} K, but to me that is not a *temperature* at all. Please read http://henk-reints.nl/astro/HR-solar-corona.pdf as well.

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[1] https://cerncourier.com/a/the-tale-of-the-hagedorn-temperature/
[2] https://en.wikipedia.org/wiki/Hagedorn_temperature (as of 2021-01-10)

We've got: $T_{rel} = \frac{2m_0c^2}{3k} \cdot \frac{(\gamma-1)}{\gamma} = \frac{2m_0c^2}{3k} \cdot \left(1-\sqrt{1-\beta^2}\right)$

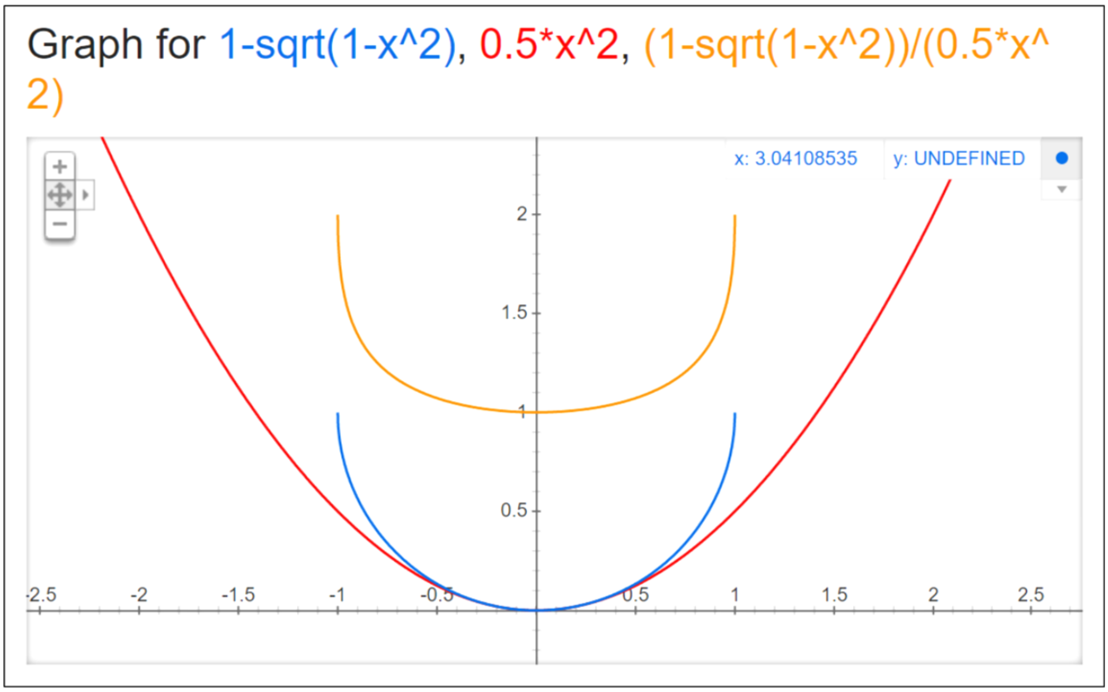
for small values of β: $T_{rel} \approx \frac{2m_0c^2}{3k} \cdot \frac{\beta^2}{2} = \frac{2}{3k} \cdot \frac{1}{2} m_0 v^2 = T_{class}$

hence: $\beta^2 = \frac{3k}{m_0c^2} T_{class}$

and: $\frac{T_{rel}}{T_{class}} = \frac{1-\sqrt{1-\beta^2}}{\beta^2/2} \approx 1 + \frac{\beta^2}{4} + \mathcal{O}(\beta^4) \;\; \therefore \;\; \frac{\Delta T}{T_{class}} = \frac{\beta^2}{4} = \frac{3k}{4c^2} \cdot \frac{T_{class}}{m_0}$

At $T = 3\,000$ K and with $m_0 = 1$ Da, this yields approximately 1 in 10^{10}, or $\Delta T = 0.31$ µK. I think this is too small to be measurable (although microkelvins near absolute zero are no problem at all).

Altogether it seems that the (to my opinion rather elegant) idea of a *relativistic Boltzmann constant* (which in fact should be called *relativistic Boltzmann factor*) does not contradict current theories. It is in agreement with the *Hagedorn temperature* and it avoids physical unlimitedness. But I picked it from thin air.



Dimensionless T_{rel}, T_{class}, and their ratio as function of the *thermal velocity* β (graph by Google).

T_{rel} is half a circle (just like the *Lorentz contraction* if drawn with an aspect ratio of 1:1), whilst T_{class} is parabolic.

Ludwig Boltzmann (1844-1906)

