In classical (Newtonian) mechanics, energy and momentum are conserved quantities. In relativistic (Einsteinian) mechanics, only their combination is conserved as the absolute value of a 4-vector named four-momentum. Below, a tilde ("~") is used to denote the latter or its components.

$$\vec{v} \coloneqq \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix}$$

$$v^2 \coloneqq v_x^2 + v_y^2 + v_z^2$$

Lorentz factor: $$\gamma^2 \coloneqq \frac{1}{1-\frac{v^2}{c^2}} = \frac{c^2}{c^2-v^2}$$

relativistic energy ($m =$ rest mass): $$\tilde{E} \coloneqq \gamma m c^2$$

four-velocity: $$\vec{\vec{v}} \coloneqq \begin{bmatrix} ic \\ v_x \\ v_y \\ v_z \end{bmatrix}$$

relativistic four-velocity: $$\vec{\tilde{u}} \coloneqq \begin{bmatrix} i\gamma c \\ \tilde{u}_x \\ \tilde{u}_y \\ \tilde{u}_z \end{bmatrix} \coloneqq \gamma \vec{\vec{v}} = \begin{bmatrix} i\gamma c \\ \gamma v_x \\ \gamma v_y \\ \gamma v_z \end{bmatrix}$$

rel. four-momentum: $$\vec{\tilde{p}} \coloneqq m\vec{\tilde{u}} = \begin{bmatrix} i\gamma mc \\ \tilde{p}_x \\ \tilde{p}_y \\ \tilde{p}_z \end{bmatrix} = \begin{bmatrix} i\tilde{E}/c \\ m\tilde{u}_x \\ m\tilde{u}_y \\ m\tilde{u}_z \end{bmatrix} = \begin{bmatrix} i\tilde{E}/c \\ \gamma m v_x \\ \gamma m v_y \\ \gamma m v_z \end{bmatrix}$$

rel. three-momentum squared: $$\tilde{p}^2 \coloneqq \tilde{p}_x^2 + \tilde{p}_y^2 + \tilde{p}_z^2 = (\gamma m v)^2$$

we find: $$\left|\vec{\tilde{p}}\right|^2 = \left(\frac{i\tilde{E}}{c}\right)^2 + \tilde{p}^2 = -\left(\frac{\tilde{E}}{c}\right)^2 + \tilde{p}^2$$

i.e.: $$-\left|\vec{\tilde{\boldsymbol{p}}}\right|^{\mathbf{2}} = \left(\frac{\tilde{\boldsymbol{E}}}{\boldsymbol{c}}\right)^{\mathbf{2}} - \tilde{\boldsymbol{p}}^{\mathbf{2}}$$

as well as: $$\left|\vec{\tilde{\boldsymbol{p}}}\right|^{\mathbf{2}} = i^2\gamma^2 m^2 c^2 + \tilde{p}_x^2 + \tilde{p}_y^2 + \tilde{p}_z^2$$

$$= \tilde{p}_x^2 + \tilde{p}_y^2 + \tilde{p}_z^2 - \gamma^2 m^2 c^2$$

$$= \gamma^2 m^2\left(v_x^2 + v_y^2 + v_z^2 - c^2\right)$$

$$= \gamma^2 m^2 (v^2 - c^2) = \frac{c^2}{c^2-v^2} m^2 (v^2 - c^2) = -\boldsymbol{m}^{\mathbf{2}}\boldsymbol{c}^{\mathbf{2}}$$

therefore: $$-\left|\vec{\tilde{\boldsymbol{p}}}\right|^{\mathbf{2}} = \boldsymbol{m}^{\mathbf{2}}\boldsymbol{c}^{\mathbf{2}} = \left(\tilde{\boldsymbol{E}}/\boldsymbol{c}\right)^{\mathbf{2}} - \tilde{\boldsymbol{p}}^{\mathbf{2}}$$

which is a **frame-independent conserved quantity** that prevails over conservation of energy and momentum as separate quantities.
It obviously & simply is **conservation of mass** (**quantitas materiæ**).

A fact that matters to matter, as a factual matter of fact.

It renders something like what was found by Pythagoras:

$$\tilde{E}^2 = (mc^2)^2 + (c\tilde{p})^2$$

The absolute value of momentum appears to be perpendicular to mass, & energy is their hypotenuse!

We found: $$\tilde{E}^2 = (mc^2)^2 + (c\tilde{p})^2$$

i.e.: $$\gamma^2 m^2 c^4 = m^2 c^4 + c^2 \gamma^2 m^2 v^2$$

divide by $m^2 c^4$: $$\gamma^2 = 1 + \frac{\gamma^2 v^2}{c^2} = 1 + \gamma^2 \beta^2$$

hence: $$\gamma^2 - 1 = \gamma^2 \beta^2$$

yielding: $$\beta^2 = \frac{\gamma^2 - 1}{\gamma^2} = 1 - \frac{1}{\gamma^2}$$ follows from conservation of mass!

The same can also be deduced from: $$\gamma = \frac{1}{\sqrt{1 - \beta^2}}$$ kinematically/geometrically using $c =$ constant

square & multiply: $$\gamma^2 (1 - \beta^2) = 1$$

distribute γ^2 : $$\gamma^2 - \gamma^2 \beta^2 = 1$$

yielding: $$\gamma^2 - 1 = \gamma^2 \beta^2$$

which renders: $$\beta^2 = \frac{\gamma^2 - 1}{\gamma^2} = 1 - \frac{1}{\gamma^2}$$

Kinetic energy: $$E_\mathrm{k} = \tilde{E} - mc^2 = \sqrt{(mc^2)^2 + (c\tilde{p})^2} - mc^2$$

so: $$\frac{E_\mathrm{k}}{mc^2} = \sqrt{1 + \frac{(c\tilde{p})^2}{(mc^2)^2}} - 1 = \sqrt{1 + \left(\frac{\tilde{p}}{mc}\right)^2} - 1 = \sqrt{1 + \left(\frac{\gamma m v}{mc}\right)^2} - 1 = \sqrt{1 + \gamma^2 \beta^2} - 1$$

i.e.: $$\boldsymbol{\mathcal{E}_\mathrm{k}} \coloneqq \frac{\boldsymbol{E_\mathrm{k}}}{\boldsymbol{mc^2}} = \sqrt{1 + \frac{\beta^2}{1 - \beta^2}} - 1 = \sqrt{\frac{1}{1 - \beta^2}} - 1 = \boldsymbol{\gamma - 1}$$

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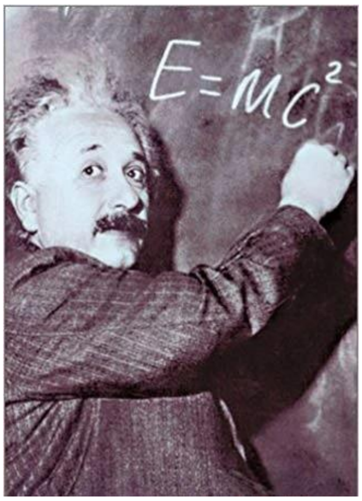

Have you ever realised that energy is a fully abstract quantity that cannot be directly observed as such? Strictly spoken, it is not a ***physical*** quantity (Greek φυσις = *form, shape, that which is natural, creature,* φυσικα = *of course, naturally*). Mathematically perfect, but if you are hit by some flying object (which *is* a natural thing), you actually ***feel*** a force $F = \dot{p}$, not the energy itself. Please also consider momentum. If this object has a greater velocity or is more massif, you ***feel*** a greater force, not the momentum itself, nor its change, called impulse. As shown above, conservation of four-momentum actually is conservation of mass (quantitas materiæ, amount of stuff). I consider mass a truly fundamental natural quantity. Matter can be touched, felt, experienced. It's what we call *something existing*. Wouldn't it be more realistic (in a physical sense) if we'd always calculate with $m = E/c^2$ instead of $E = mc^2$?

