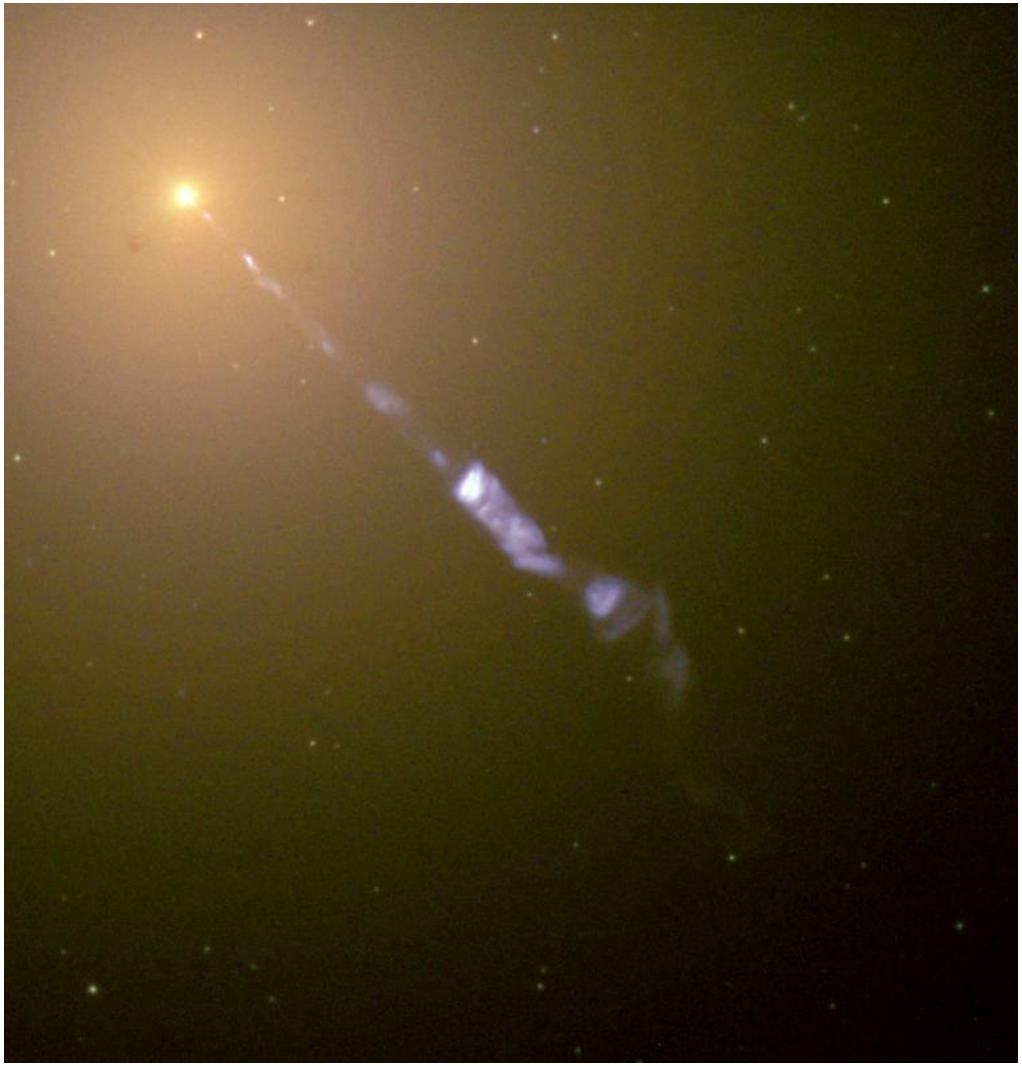
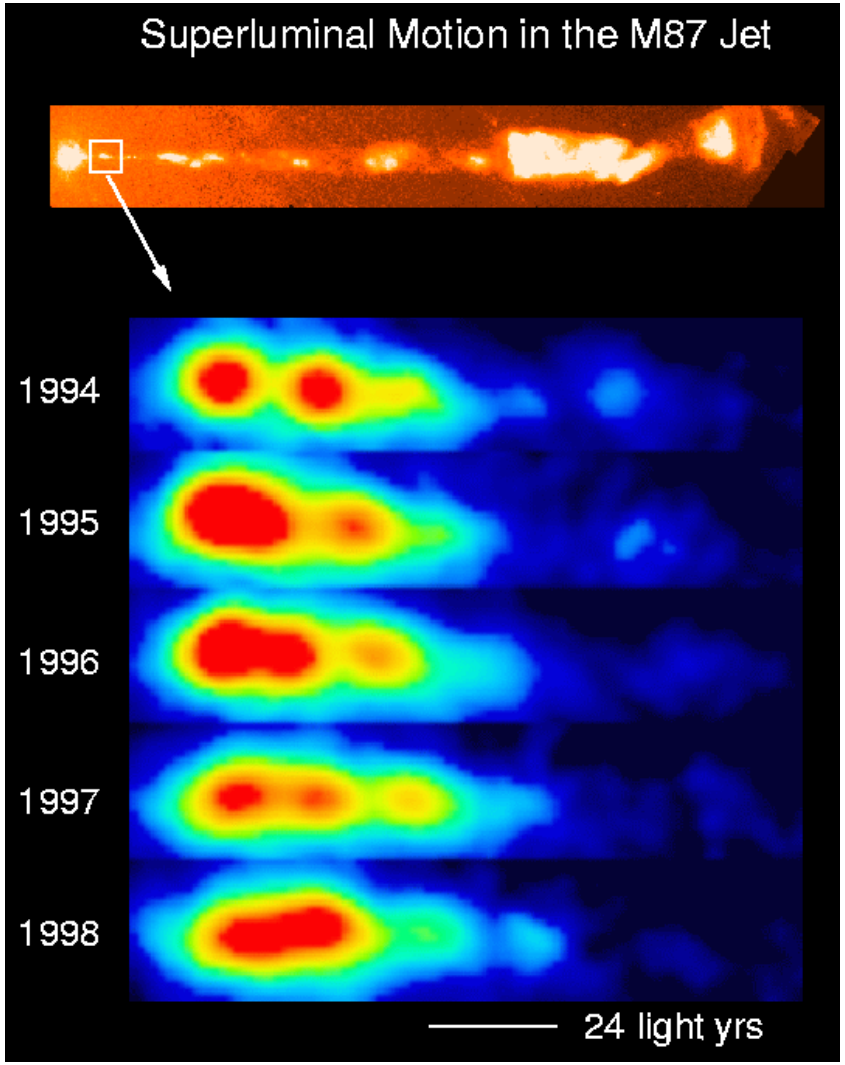
The apparently superluminal jet of M87:



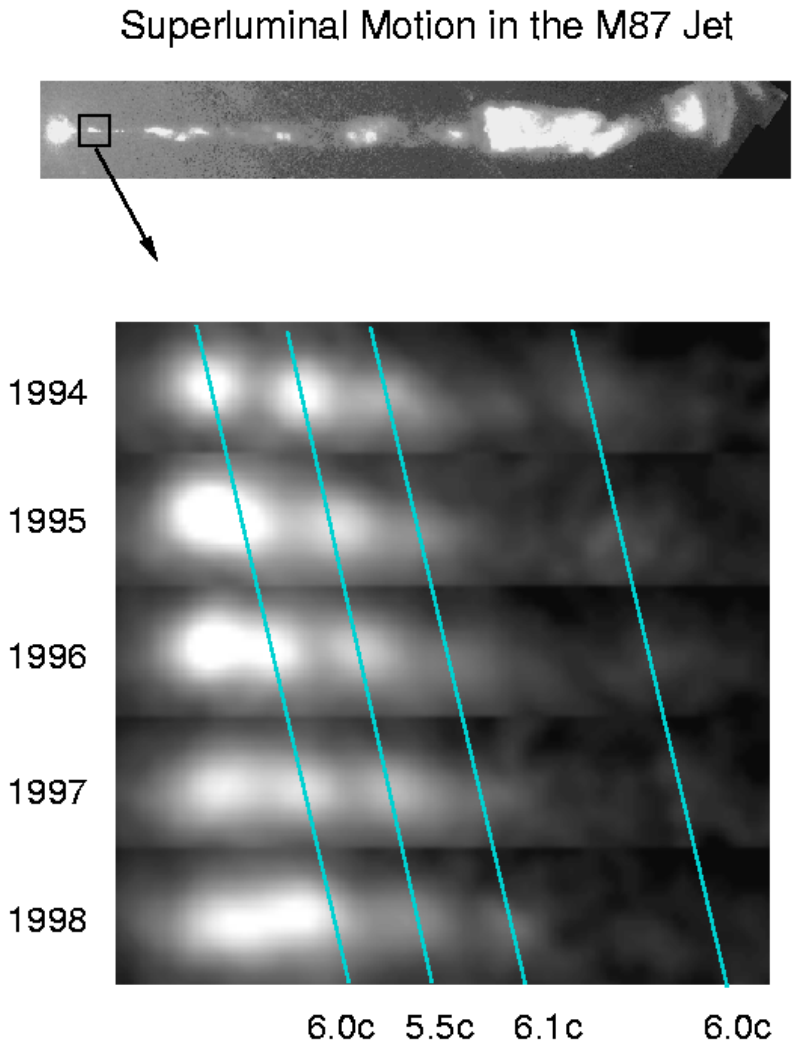
https://stsci-opo.org/STScI-01EVVP5H3659JD1BKBBWAX5ES5.jpg
via https://hubblesite.org/contents/media/images/2000/20/968-Image.html



Sequence of Hubble images showing apparent motion at six times the speed of light in the galaxy M87. TOP PANEL: Hubble image showing jet streaming out from the galaxy's nucleus [bright round region at far left]. The jet is about 5000 light years long, and the box indicates where the superluminal motions were seen. BOTTOM PANEL: Sequence of Hubble images showing motion at six times the speed of light. The scale bar is 24 light years long, which is approximately the distance the clouds appear to travel in 4 years. The images were made between 1994 and 1998 with the Faint Object Camera on the Hubble Space Telescope. PHOTO CREDIT: John Biretta, Space Telescope Science Institute.

https://www.stsci.edu/ftp/science/m87/color5.gif

https://www.stsci.edu/ftp/science/m87/m87.html

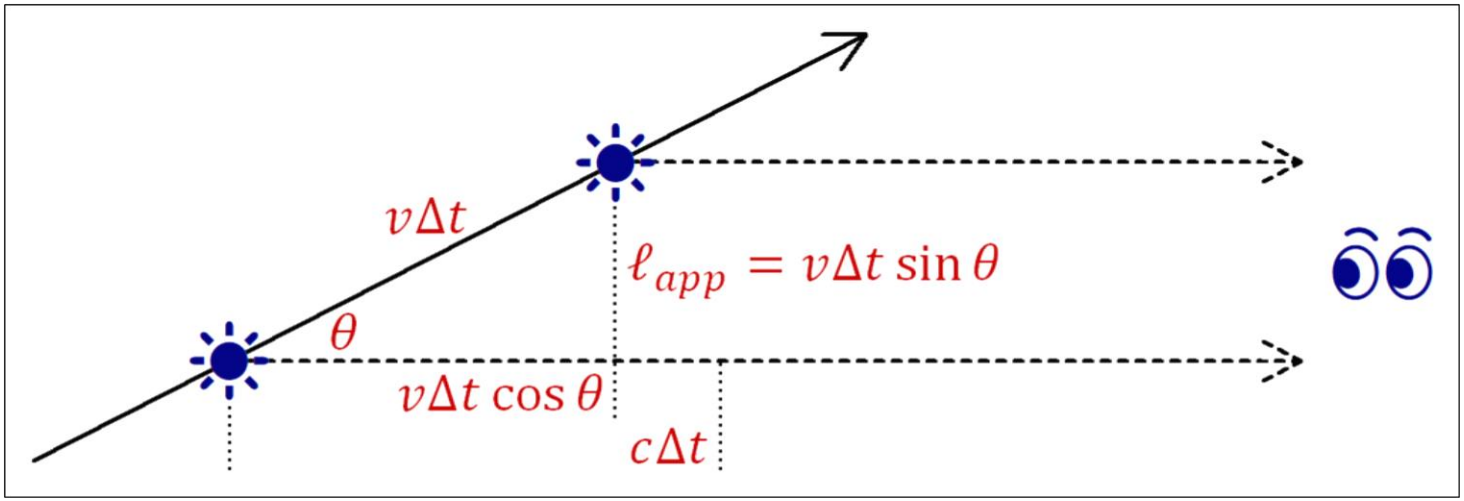


Sequence of Hubble images showing apparent motion at six times the speed of light in the galaxy M87. TOP PANEL: Hubble image showing jet streaming out from the galaxy's nucleus [bright round region at far left]. The jet is about 5000 light years long, and the box indicates where the superluminal motions were seen. BOTTOM PANEL: Sequence of Hubble images showing motion at six times the speed of light. The slanting lines track the moving features, and the speeds are given in units of the velocity of light "c." The images were made between 1994 and 1998 with the Faint Object Camera on the Hubble Space Telescope. PHOTO CREDIT: John Biretta, Space Telescope Science Institute.

https://www.stsci.edu/ftp/science/m87/bw3.gif

https://www.stsci.edu/ftp/science/m87/m87.html

Suppose a (part of a) jet is coming more or less
(but more more than less) towards us at a velocity $\boldsymbol{v}$ and an angle $\boldsymbol{\theta}$:



then the observed time between two light emissions with an interval of Δt is:

$$\Delta t_{app} = \frac{c\Delta t - v\Delta t \cos\theta}{c} = \Delta t(1 - \beta \cos\theta)$$

hence the apparent transverse velocity equals:

$$v_{app} = \frac{\ell_{app}}{\Delta t_{app}} = \frac{v\Delta t \sin\theta}{\Delta t(1-\beta\cos\theta)} \quad \therefore \quad \boldsymbol{\beta_{app} = \frac{\beta \sin\theta}{1-\beta\cos\theta}}$$

Apparent transverse velocity:

$$\boldsymbol{\beta_{app} = \frac{\beta \sin\theta}{1-\beta\cos\theta}}$$

then:

$$\frac{d\beta_{app}}{d\theta} = \frac{(1-\beta\cos\theta)\cdot(\beta\cos\theta)-(\beta\sin\theta)\cdot(\beta\sin\theta)}{(1-\beta\cos\theta)^2} = \frac{\beta\cos\theta-\beta^2\cos^2\theta-\beta^2\sin^2\theta}{(1-\beta\cos\theta)^2} = \frac{\beta\cos\theta-\beta^2}{(1-\beta\cos\theta)^2}$$

$$\boldsymbol{= \beta\frac{\cos(\theta)-\beta}{(1-\beta\cos\theta)^2}}$$

Maximum for given β is at:

$$\boldsymbol{\cos\theta = \beta \quad \therefore \quad \theta = \arccos\beta}$$

The value of this maximum is:

$$\boldsymbol{\beta_{app,max}} = \frac{\beta\sin(\arccos\beta)}{1-\beta\cos(\arccos\beta)} = \frac{\beta\sqrt{1-\beta^2}}{1-\beta^2} = \frac{\beta}{\sqrt{1-\beta^2}} \boldsymbol{= \beta\gamma}$$

Now $\boldsymbol{\gamma}$ becomes very large when $\boldsymbol{\beta}$ approaches $\mathbf{1}$,

so a *sub*luminal jet coming towards us can show an apparent *super*luminal transverse velocity.

Critical point for apparent superluminality:

$$\beta_{app} = \frac{\beta \sin\theta}{1-\beta\cos\theta} = 1 \quad \therefore \quad \beta\sin\theta = 1-\beta\cos\theta$$

$$\therefore \quad \sin\theta + \cos\theta = \frac{1}{\beta} = \frac{1}{\cos\theta} \quad \therefore \quad \sqrt{2}\sin\left(\theta+\frac{\pi}{4}\right) = \frac{1}{\cos\theta}$$

$$\therefore \quad \sin\left(\theta+\frac{\pi}{4}\right)\cos\theta = \tfrac{1}{2}\sqrt{2}$$

Meaningful solution:

$$\boldsymbol{\theta = \frac{\pi}{4} \cong 45^\circ} \quad \therefore \quad \boldsymbol{\beta = \tfrac{1}{2}\sqrt{2}} \quad \therefore \quad \boldsymbol{\gamma = \sqrt{2}}$$

See next page for a more complete overview, from which I estimate that:

the apparently superluminal portion of the **M87** jet probably comes towards us at $\boldsymbol{\beta \approx 0.99}$ & $\boldsymbol{\theta \approx 14^\circ}$.

Ceterum censeo superluminalitatem esse delendam.

Apparent transverse β:

beta jet	theta 10°	20°	30°	40°	50°	60°	70°	80°	90°	
0.99:	**6.27**	**6.87**	**5.86**	**4.86**	**4.07**	**3.47**	**3.00**	**2.63**	**2.33**	**2.09**	**1.88**	**1.70**	**1.54**	**1.41**	**1.29**	**1.18**	**1.08**	.99
0.98:	**3.60**	**4.88**	**4.75**	**4.24**	**3.70**	**3.24**	**2.85**	**2.53**	**2.26**	**2.03**	**1.83**	**1.66**	**1.52**	**1.39**	**1.27**	**1.16**	**1.07**	.98
0.97:	**2.51**	**3.77**	**3.98**	**3.75**	**3.39**	**3.03**	**2.71**	**2.43**	**2.18**	**1.97**	**1.79**	**1.63**	**1.49**	**1.36**	**1.25**	**1.15**	**1.06**	.97
0.96:	**1.92**	**3.05**	**3.42**	**3.35**	**3.12**	**2.85**	**2.58**	**2.33**	**2.11**	**1.92**	**1.75**	**1.60**	**1.46**	**1.34**	**1.23**	**1.13**	**1.04**	.96
0.95:	**1.54**	**2.56**	**2.99**	**3.03**	**2.89**	**2.68**	**2.46**	**2.24**	**2.05**	**1.87**	**1.71**	**1.57**	**1.44**	**1.32**	**1.22**	**1.12**	**1.03**	.95
0.94:	**1.29**	**2.20**	**2.64**	**2.76**	**2.68**	**2.53**	**2.34**	**2.16**	**1.98**	**1.82**	**1.67**	**1.54**	**1.41**	**1.30**	**1.20**	**1.11**	**1.02**	.94
0.93:	**1.10**	**1.92**	**2.37**	**2.52**	**2.50**	**2.39**	**2.24**	**2.08**	**1.92**	**1.77**	**1.63**	**1.51**	**1.39**	**1.28**	**1.18**	**1.09**	**1.01**	.93
0.92:	.96	**1.70**	**2.14**	**2.32**	**2.34**	**2.26**	**2.14**	**2.00**	**1.86**	**1.72**	**1.60**	**1.48**	**1.36**	**1.26**	**1.17**	**1.08**	**1.00**	.92
0.91:	.85	**1.52**	**1.95**	**2.15**	**2.19**	**2.15**	**2.05**	**1.93**	**1.80**	**1.68**	**1.56**	**1.45**	**1.34**	**1.24**	**1.15**	**1.06**	.98	.91
0.90:	.76	**1.37**	**1.78**	**2.00**	**2.06**	**2.04**	**1.96**	**1.86**	**1.75**	**1.64**	**1.52**	**1.42**	**1.32**	**1.22**	**1.13**	**1.05**	.97	.90
0.89:	.68	**1.25**	**1.64**	**1.86**	**1.94**	**1.94**	**1.88**	**1.80**	**1.70**	**1.59**	**1.49**	**1.39**	**1.29**	**1.20**	**1.12**	**1.04**	.96	.89
0.88:	.62	**1.15**	**1.52**	**1.74**	**1.84**	**1.85**	**1.81**	**1.74**	**1.65**	**1.55**	**1.46**	**1.36**	**1.27**	**1.18**	**1.10**	**1.02**	.95	.88
0.87:	.57	**1.05**	**1.41**	**1.63**	**1.74**	**1.76**	**1.74**	**1.68**	**1.60**	**1.51**	**1.42**	**1.33**	**1.25**	**1.16**	**1.08**	**1.01**	.94	.87
0.86:	.52	.98	**1.31**	**1.53**	**1.65**	**1.68**	**1.67**	**1.62**	**1.55**	**1.47**	**1.39**	**1.31**	**1.22**	**1.14**	**1.07**	**1.00**	.93	.86
0.85:	.48	.91	**1.23**	**1.44**	**1.56**	**1.61**	**1.61**	**1.57**	**1.51**	**1.44**	**1.36**	**1.28**	**1.20**	**1.13**	**1.05**	.98	.91	.85
0.84:	.45	.84	**1.15**	**1.36**	**1.49**	**1.54**	**1.54**	**1.51**	**1.46**	**1.40**	**1.33**	**1.25**	**1.18**	**1.11**	**1.04**	.97	.90	.84
0.83:	.42	.79	**1.08**	**1.29**	**1.42**	**1.48**	**1.49**	**1.46**	**1.42**	**1.36**	**1.30**	**1.23**	**1.16**	**1.09**	**1.02**	.96	.89	.83
0.82:	.39	.74	**1.02**	**1.22**	**1.35**	**1.41**	**1.43**	**1.42**	**1.38**	**1.33**	**1.27**	**1.20**	**1.14**	**1.07**	**1.01**	.94	.88	.82
0.81:	.37	.70	.96	**1.16**	**1.29**	**1.36**	**1.38**	**1.37**	**1.34**	**1.29**	**1.24**	**1.18**	**1.12**	**1.05**	.99	.93	.87	.81
0.80:	.34	.65	.91	**1.10**	**1.23**	**1.30**	**1.33**	**1.33**	**1.30**	**1.26**	**1.21**	**1.15**	**1.10**	**1.03**	.97	.91	.86	.80
0.79:	.32	.62	.86	**1.05**	**1.18**	**1.25**	**1.28**	**1.29**	**1.27**	**1.23**	**1.18**	**1.13**	**1.07**	**1.02**	.96	.90	.85	.79
0.78:	.30	.58	.82	**1.00**	**1.12**	**1.20**	**1.24**	**1.25**	**1.23**	**1.20**	**1.16**	**1.11**	**1.05**	**1.00**	.94	.89	.83	.78
0.77:	.29	.55	.78	.95	**1.08**	**1.16**	**1.20**	**1.21**	**1.20**	**1.17**	**1.13**	**1.08**	**1.03**	.98	.93	.88	.82	.77
0.76:	.27	.52	.74	.91	**1.03**	**1.11**	**1.15**	**1.17**	**1.16**	**1.14**	**1.10**	**1.06**	**1.01**	.97	.91	.86	.81	.76
0.75:	.26	.50	.70	.87	.99	**1.07**	**1.12**	**1.13**	**1.13**	**1.11**	**1.08**	**1.04**	**1.00**	.95	.90	.85	.80	.75
0.74:	.25	.47	.67	.83	.95	**1.03**	**1.08**	**1.10**	**1.10**	**1.08**	**1.05**	**1.02**	.98	.93	.88	.84	.79	.74
0.73:	.23	.45	.64	.80	.91	.99	**1.04**	**1.06**	**1.07**	**1.05**	**1.03**	**1.00**	.96	.91	.87	.82	.78	.73
0.72:	.22	.43	.61	.76	.88	.96	**1.01**	**1.03**	**1.04**	**1.03**	**1.00**	.97	.94	.90	.85	.81	.77	.72
0.71:	.21	.41	.58	.73	.84	.92	.97	**1.00**	**1.01**	**1.00**	.98	.95	.92	.88	.84	.80	.75	.71
0.70:	.20	.39	.56	.70	.81	.89	.94	.97	.98	.97	.96	.93	.90	.86	.83	.78	.74	.70
0.69:	.19	.37	.54	.67	.78	.86	.91	.94	.95	.95	.94	.91	.88	.85	.81	.77	.73	.69
0.68:	.18	.36	.51	.64	.75	.83	.88	.91	.93	.93	.91	.89	.86	.83	.80	.76	.72	.68
0.67:	.18	.34	.49	.62	.72	.80	.85	.88	.90	.90	.89	.87	.85	.82	.78	.75	.71	.67
0.66:	.17	.33	.47	.59	.69	.77	.82	.86	.88	.88	.87	.85	.83	.80	.77	.73	.70	.66
0.65:	.16	.31	.45	.57	.67	.74	.80	.83	.85	.86	.85	.83	.81	.79	.75	.72	.69	.65

SuperluminahahaHaHaHAHA...

Please tell me another one!