Gravitational potential energy:
gravitational potential:
Escape velocity:

Kinetic energy of free fall:
free fall from $\infty$, starting with:
specific kinetic energy:
Free fall velocity:

## Conservation of energy:

therefore (obviously):

This renders:
therefore:

From SR, we have:
and from GR:

$$
\begin{align*}
& V-V_{\infty}=-\frac{G M m}{r}  \tag{1}\\
& U=\frac{V-V_{\infty}}{m}=\frac{-G M}{r}  \tag{2}\\
& v_{\mathrm{esc}}=\sqrt{\frac{2 G M}{r}} \quad \Leftrightarrow v_{\mathrm{esc}}^{2}=-2 U \quad \therefore \frac{v_{\mathrm{esc}}^{2}}{c^{2}}=\frac{-2 U}{c^{2}}=\frac{2 G M}{r c^{2}} \tag{3}
\end{align*}
$$

$$
\begin{align*}
& T=\frac{1}{2} m v_{\mathrm{ff}}^{2}  \tag{4}\\
& v_{\mathrm{ff} @}=0 \therefore T_{\infty}=0  \tag{5}\\
& S=\frac{T}{m}=\frac{1}{2} v_{\mathrm{ff}}^{2} \tag{6}
\end{align*}
$$

$T=V_{\infty}-V$
$\therefore v_{\mathrm{ff}}^{2}=2 S$
$\therefore \frac{v_{\mathrm{ff}}^{2}}{c^{2}}=\frac{2 S}{c^{2}}$
$\frac{2 S}{c^{2}}=\frac{-2 U}{c^{2}}$
$\therefore S=-U \quad \therefore v_{\mathrm{ff}}^{2}=v_{\mathrm{esc}}^{2}$
$\sqrt{1-\frac{2 S}{c^{2}}}=\sqrt{1+\frac{2 U}{c^{2}}}$
$\sqrt{1-\frac{v_{\mathrm{ff}}^{2}}{c^{2}}}=\sqrt{1-\frac{v_{\mathrm{scc}}^{2}}{c^{2}}}=\sqrt{1-\frac{2 G M}{r c^{2}}}$
$\sqrt{1-\frac{v_{\mathrm{ff}}^{2}}{c^{2}}}=\sqrt{1-\beta_{\mathrm{ff}}^{2}}=\frac{1}{\gamma}$
$\sqrt{1-\frac{v_{\mathrm{esc}}^{2}}{c^{2}}}=\sqrt{1-\beta_{\mathrm{esc}}^{2}}=\sqrt{1-\frac{2 G M}{r c^{2}}}=\sqrt{1-\frac{r_{\mathrm{s}}}{r}}$
Yielding (with $\beta_{\mathrm{ff}}=-\beta_{\mathrm{esc}}$ ):

$$
\begin{equation*}
\frac{1}{\gamma}=\sqrt{1-\frac{r_{s}}{r}} \tag{14}
\end{equation*}
$$

and

$$
\beta_{\mathrm{ff}}^{2}=\frac{r_{\mathrm{s}}}{r}
$$

The reciprocal Lorentz factor of a freely falling body (which has accelerated to its free fall velocity) equals the "Schwarzschild factor" that arises from the gravitational field at its current location. This simply follows from the conservation of energy.

Potential energy is due to gravitation \& acceleration turns it into kinetic energy.
Einstein's equivalence principle: a gravitational field and acceleration are indistinguishable.

## Conservation of energy and Einstein's equivalence principle apparently are the very same.



## Is this a new insight?

Google renders not much when searching for:
equivalence principle conservation energy
I found: https://arxiv.org/abs/gr-qc/0409121
(as of 2021)

He actually said (to Louis de Broglie): All physical theories, their mathematical expression apart, ought to lend themselves to so simple a description that even a child could understand them.

