In http://henk-reints.nl/astro/HR-Relativity-and-curvature-of-spacetime.pdf, I explain:

> When accelerating, you build up a velocity with respect to the uniform motion you would have had without acceleration. Acceleration is caused by a force, so when you are subject to some force, you build up a velocity with respect to the inert motion you would have had without that force.
> Einstein's happiest thought of his life was: *In free fall you don't feel your own weight. During a free fall in a gravitational field, you can very well substantiate that you're at rest or in inert motion.*
> To be (radially) stationary in a gravitational field, an upward force is required that prevents your free fall. In practice, it will be exerted by the resilience of the (nearly) incompressible matter forming a planet. It *is* what you feel with your bottom when sitting on the floor or on a chair.
> The (pseudo) acceleration caused by this force yields an effective (gravitational pseudo) velocity (=: "*gravocity*") w.r.t. your inert motion, which is the free fall. Obviously, this *gravocity* equals the escape velocity. Since the latter does not actually change, the *gravocity* is constant, despite a continuous force being exerted. Whilst sitting on a chair, you effectively have an upward velocity of 11.2 km/s! Like any velocity, the *gravocity* causes kinematic time dilation, including all of its consequences. We call it gravitational time dilation. *That* is the so-called Equivalence Principle[1].

For the perihelion precession[2], Einstein derived: $\varepsilon = 24\pi^3 \frac{a^2}{T^2c^2(1-e^2)}$

First of all, I rename & rewrite it: $\Delta\varphi_{/\mathrm{orb}} = \frac{6\pi\cdot(2\pi a)^2}{c^2T^2(1-e^2)}$

where the "/orb" suffix should better clarify that it is some angle per orbit. It obviously yields a non-zero positive value for any $e < 1$, i.e. even for circular orbits where $e = 0$, so we should rather speak of *orbit* precession than *perihelion* precession. We can simplify Einstein's result by recognising that $2\pi a/T$ is the orbital velocity it would have at a distance a & that c makes it dimensionless,

hence: $\frac{2\pi a}{cT} = \beta_a$

yielding: $\Delta\varphi_{/\mathrm{orb}} = \frac{6\pi\beta_a^2}{(1-e^2)}$ (isn't this way more fathomable?)

We also have (via Kepler 3): $\beta_a = \frac{\sqrt{GM/a}}{c} = \sqrt{\frac{GM}{ac^2}} = \sqrt{\frac{r_S}{2a}} \therefore \beta_a^2 = \frac{r_S}{2a}$

which renders: $\Delta\varphi_{/\mathrm{orb}} = \frac{3\pi r_S}{a(1-e^2)}$

It happens to be that: $\frac{1}{a(1-e^2)} = \frac{2a}{2\cdot a(1+e)\cdot a(1-e)} = \frac{r_a+r_p}{2r_ar_p} = \frac{1}{2}\left(\frac{1}{r_p}+\frac{1}{r_a}\right)$

equals the average *proximity* of periapsis & apoapsis. Geometrically, it *is* the *mean proximity* of an entire orbit. Its reciprocal equals the *semi latus rectum*, denoted $r \coloneqq a(1-e^2)$, which in turn equals the *radius of curvature* in both periapsis and apoapsis. For a circular orbit, r would simply equal the *radius*. We divide it by the Schwarzschild radius in order to make it dimensionless:

$$\rho \coloneqq \frac{r}{r_S} = \frac{a(1-e^2)}{r_S}$$

so Einstein's result actually is: $\Delta\varphi_{/\mathrm{orb}} = \frac{3\pi}{\rho}$ (ain't that nice?)

We will now restrict ourselves to Special Relativity. ***No*** tensor calculus, but we will include the aforementioned *gravocity* in our calculations.

The orbital velocity is: $v_{\mathrm{orb}} = \sqrt{\frac{GM}{r}} \therefore \beta_{\mathrm{orb}} = \sqrt{\frac{r_S}{2r}} = \sqrt{\frac{1}{2\rho}}$

yielding a Lorentz[3] factor of: $\gamma_{\mathrm{orb}} = 1/\sqrt{1-\beta_{\mathrm{orb}}^2} = 1/\sqrt{1-\frac{1}{2\rho}}$

[1] See http://henk-reints.nl/astro/HR-Equivalence-principle.pdf

[2] See https://articles.adsabs.harvard.edu/pdf/1915SPAW.......831E

[3] Prof. Hendrik Antoon Lorentz was a Dutchman and so am I (and in my passport I'm *Hendrik* as well); please stress the 1st syllable: ***LO**rentz* and **not**: Lo***RENTZ*** which sounds "unDutch" (*Lawrence* = same name).

The escape velocity equals: $v_{\text{esc}} = \sqrt{\frac{2GM}{r}} \therefore \beta_{\text{esc}} = \sqrt{\frac{r_S}{r}} = \sqrt{\frac{1}{\rho}}$

This equals the *gravocity*,
which causes time dilation by: $\gamma_{\text{grav}} = 1/\sqrt{1-\beta_{\text{esc}}^2} = 1/\sqrt{1-\frac{1}{\rho}}$ (Equiv. Pr.)

Wouldn't an inert and stationary distant observer measure time stretching of the orbital period?

This would of course yield: $T_{\text{dist}} = T_{\text{prop}} \cdot \gamma_{\text{orb}}\gamma_{\text{grav}}$

and therefore: $T_{\text{prop}} = T_{\text{dist}} \cdot \gamma_{\text{orb}}^{-1}\gamma_{\text{grav}}^{-1}$

We find: $\gamma_{\text{orb}}^{-1}\gamma_{\text{grav}}^{-1} = \sqrt{\left(1-\frac{1}{2\rho}\right)\left(1-\frac{1}{\rho}\right)} = \sqrt{\frac{2\rho^2-3\rho+1}{2\rho^2}}$

which has a Laurent series at $\rho = \infty$: $\mathbf{1 - \frac{3}{4\rho}} - \frac{1}{32\rho^2} - \frac{3}{128\rho^3} - \frac{37}{2048\rho^4} - \mathcal{O}\left(\frac{1}{\rho^5}\right)$

In first order, we would have: $T_{\text{prop}} = T_{\text{dist}}\left(1-\frac{3}{4\rho}\right)$

So the planet perceives a shortage of: $\Delta T = \frac{3T_{\text{dist}}}{4\rho}$

which it can compensate by orbiting a bit further. *This* could be the ~~perihelion~~ ***orbit*** precession.

Per orbit, this renders: $\Delta\boldsymbol{\varphi}_{/\text{orb}} = 2\pi\frac{\Delta T}{T_{\text{dist}}} = \boldsymbol{\frac{3\pi}{2\rho}}$

which is only half of Einstein's result ☹. Close, but no ~~cigar~~, ~~broccoli~~, icecream.

But doesn't *time dilation* cause *length contraction* as well? Doesn't the planet perceive orbital path length contraction by the very same γ_{orb}^{-1} , thus shortening the orbital period by this factor once again?

And wouldn't *gravitational time dilation* due to the *gravocity* contract the radius, hence the circumference as well? Wouldn't this yield a similar result as the orbital velocity? Shouldn't we square both Lorentz factors?

We would obtain: $T_{\text{prop}} = T_{\text{dist}} \cdot \gamma_{\text{orb}}^{-2}\gamma_{\text{grav}}^{-2}$

and: $\gamma_{\text{orb}}^{-2}\gamma_{\text{grav}}^{-2} = \left(1-\frac{1}{2\rho}\right)\left(1-\frac{1}{\rho}\right) = 1 - \frac{3}{2\rho} + \frac{1}{2\rho^2}$

yielding: $T_{\text{prop}} = T_{\text{dist}}\left(1-\frac{3}{2\rho}+\frac{1}{2\rho^2}\right)$

hence: $\Delta T = \frac{3T_{\text{dist}}}{2\rho} - \frac{T_{\text{dist}}}{2\rho^2}$

which renders per orbit: $\Delta\boldsymbol{\varphi}_{/\text{orb}} = 2\pi\frac{\Delta T}{T_{\text{dist}}} = \boldsymbol{\frac{3\pi}{\rho} - \frac{\pi}{\rho^2}}$

where: $\rho =$ dimensionless *semi latus rectum* $= \rho_{\text{p}}(1+e)$

In first order, it obviously is: $\Delta\boldsymbol{\varphi}_{/\text{orb}} = \boldsymbol{\frac{3\pi}{\rho}}$ **BINGO!**

Lieber Albert, do you see we have just derived an *exact* solution of the ~~perihelion~~ ***orbit*** precession instead of merely a first order approximation? And that no tensor calculus was needed?

Can/should we conclude orbits precess due to their period being frame independent?

I have to admit I in fact used this implicit ***assumption*** as a premise, but isn't rotation considered something absolute, i.e. frame independant? Doesn't this at least *suggest* invariance or frame independency of the orbital period? It also makes the orbital circumference frame independant, since v is the same for both.

Isn't this way more plausible than that silly duration equality in the Bernoulli "explanation" of how a wing works?

Time dilation causes the *proper orbital period* to be *shorter* than what is observed by a stationary observer at (or "near") infinity, who perceives *time stretching*. It also causes a *contracted proper orbital path length* in comparison with what this distant observer sees, yielding the same orbital period shortening once again. This applies to both the orbital velocity and the *gravocity*. From the planet's point of view, the orbit has not yet completed, so it orbits a bit further.

Dear reader, do you now finally ***UNDERSTAND*** the ~~perihelion~~ ***orbit*** precession ***in a physical way***, instead of merely being aware of a rather complicated (but correct!) mathematical derivation?

The definition of genius is taking the complex and making it simple.
Albert Einstein: "I never said that".

(HR: *No, I'm not arrogant, just very good...*)

For Mercury orbiting the sun, we have:

Schwarzschild radius of sun:	$r_{S\odot} \approx 2953.34$ m
Mercury's perihelion distance:	$r_p \approx 0.307499$ au $\approx 4.60012 \times 10^{10}$ m
excentricity:	$e = 0.205630$
orbital period:	$T = 87.9691$ days $(= T_{dist})$
orbits per Julian century (36525 days):	$N = 36525/T = 415.203$

From which we find:

semi latus rectum:	$r = r_p(1+e) \approx 5.54604 \times 10^{10}$ m
dimensionless:	$\rho = r/r_{S\odot} \approx 1.87789 \times 10^7$
first order precession per orbit:	$\Delta\varphi_{/orb} = 3\pi/\rho \approx 5.01882 \times 10^{-7}$ rad
per century:	$\Delta\varphi_{/cent} = N\Delta\varphi_{/orb} \approx 2.08383 \times 10^{-4}$ rad
which equals:	$\boldsymbol{\Delta\varphi_{/cent} = 42''.9821}$

It should be obvious that orbit precession is a repair of the *orbital period shortage* w.r.t. the Newtonian equivalent of an orbiting body. Could such a shortage exceed one orbit per orbit? Of course not!

Then it must be that: $\Delta\varphi_{/orb} = \frac{3\pi}{\rho} \le 2\pi \therefore \boldsymbol{\rho \ge \frac{3}{2}}$. Doesn't this equal the photon sphere?

But this is merely a first order approximation.

This exact solution would be restricted by: $\frac{\Delta T}{T} = \frac{3}{2\rho} - \frac{1}{2\rho^2} \le 1 \therefore 3\rho - 1 \le 2\rho^2$

so: $2\rho^2 - 3\rho + 1 \ge 0 \therefore \rho_{1,2} = \frac{3 \pm \sqrt{9-8}}{4} = \left\{\frac{1}{2}, 1\right\}$

hence: $\rho \le \frac{1}{2}$ (which we can ignore) or: $\boldsymbol{\rho \ge 1}$.

This is of course not the photon sphere, but the Schwarzschild radius. Might the standard BH equation be flawed? In http://henk-reints.nl/astro/HR-fall-into-black-hole-slides.pdf , I substantiate *it is* flawed!

In the same document, I substantiate that — based on the conservation of energy & using relativistic kinetic energy — a radial free fall should proceed according to:

$$(\gamma - 1)mc^2 = GMm/r \therefore \gamma = 1 + r_S/2r = 1 + 1/2\rho \therefore \boldsymbol{\gamma_{ff} = (2\rho + 1)/2\rho}$$

The Equivalence Principle (which ***is*** the conservation of energy!) says the very same $\boldsymbol{\gamma_{ff}}$ should then be used for gravitational time dilation and length contraction, instead of $1/\sqrt{1 - 1/\rho}$. Via their series expansions we find they are equal in first order.

It must therefore be that: $\gamma_{grav} = \gamma_{ff} = \frac{2\rho+1}{2\rho}$

& then the orbital precession becomes: $\frac{\Delta\varphi_{/orb}}{2\pi} = 1 - \gamma_{grav}^{-2}\gamma_{orb}^{-2} = 1 - \frac{(2\rho)^2}{(2\rho+1)^2}\left(1 - \beta_{orb}^2\right)$

We also have: $1 - \beta_{orb}^2 = \gamma_{orb}^{-2} = 1 - \frac{1}{2\rho} = \frac{2\rho-1}{2\rho}$

hence: $\frac{\Delta\varphi_{/orb}}{2\pi} = 1 - \frac{(2\rho)^2}{(2\rho+1)^2} \cdot \frac{2\rho-1}{2\rho} = 1 - \frac{2\rho(2\rho-1)}{(2\rho+1)^2}$

yielding the exact solution: $\boldsymbol{\frac{\Delta\varphi_{/orb}}{2\pi}} = \frac{(2\rho+1)^2 - 2\rho(2\rho-1)}{(2\rho+1)^2} = \frac{4\rho^2+4\rho+1 - 4\rho^2+2\rho}{(2\rho+1)^2} = \boldsymbol{\frac{6\rho+1}{(2\rho+1)^2}}$

which expands to: $\frac{\Delta\varphi_{/orb}}{2\pi} = \frac{3}{2\rho} - \frac{5}{4\rho^2} + \frac{7}{8\rho^3} - \frac{9}{16\rho^4} + \frac{11}{32\rho^5} + \mathcal{O}\left(\frac{1}{\rho^6}\right)$

hence: $\boldsymbol{\Delta\varphi_{/orb} = \frac{3\pi}{\rho} - \frac{5\pi}{2\rho^2} + \frac{7\pi}{4\rho^3} - \frac{9\pi}{8\rho^4} + \frac{11\pi}{16\rho^5} + \mathcal{O}\left(\frac{1}{\rho^6}\right)}$

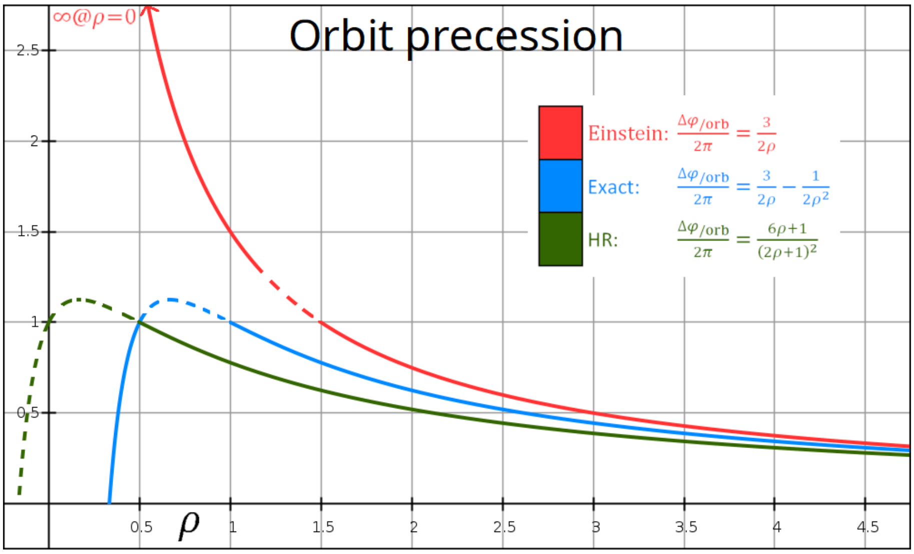
so in 1st order it (of course) equals what we've found before.

We now also have: $\frac{\Delta\varphi_{/orb}}{2\pi} = \frac{6\rho+1}{(2\rho+1)^2} \le 1 \to \rho \ge \frac{1}{2}$ (i.e. no photon sphere)

as well as: $\rho = 1 \quad \to \frac{\Delta\varphi_{/orb}}{2\pi} = \frac{7}{9} < 1$

We must now choose

either: $\boldsymbol{\frac{\Delta\varphi_{/orb,HR}}{2\pi} = \frac{6\rho+1}{(2\rho+1)^2}}$ or: $\left(\frac{\Delta\varphi_{/orb,AE}}{2\pi} = \frac{3}{2\rho}\right) - \frac{1}{2\rho^2} = \frac{3\rho-1}{2\rho^2} = \frac{6\rho-2}{(2\rho)^2}$



Orbit precession as fraction of full orbit per orbit

Distinction can only be made quite near the gravitating mass.

S2 orbits Sgr A*

at $\rho = a_{S2}(1 - e_{S2}^2)/r_{S,SgrA^*} \approx 2638$

yielding:

$\frac{3\pi}{\rho}$	$\approx 736''.92$
$\frac{3\pi}{\rho} - \frac{\pi}{\rho^2}$	$\approx 736''.83$
$\frac{2\pi(6\rho+1)}{(2\rho+1)^2}$	$\approx 736''.69$
Arcmin high:	$\approx 12'.282$
low:	$\approx 12'.278$
Observed[4]:	$\approx 12'$

All physical theories, their mathematical expression apart, ought to lend themselves to so simple a description that even a child could understand them.
— Albert Einstein[5] —

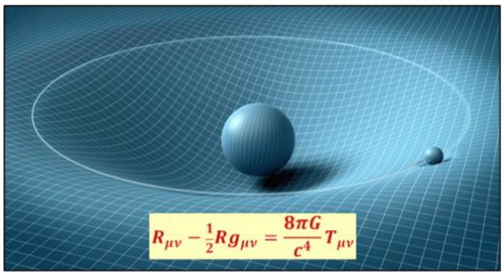
To resolve the orbit precession, Einstein needed the behemoth of tensor calculus including nasty things named Christoffel symbols, made two hard-to-fathom approximations, encountered an elliptic integral and, ultimately, he gave only an *approximative description* in the form of a *mathematical expression*[2@p2].

This very document gives a *full explanation*, yielding *physical* insight, comprehensible to a child.
It also gives two *exact* solutions (of which obviously only one can be correct and of course that's mine 😋).

Does Mercury
(the Roman god of financial gain, commerce, eloquence, messages, communication, travellers, boundaries, luck, trickery, and thieves)
really "prove" general relativity?

Does GR *explain* or merely *describe* gravitation in ***A*** possible way?
HOW ~~on earth~~ in the cosmos does mass curve spacetime or attract another mass?

Please note: I'm not saying GR is wrong, on the contrary, but it should not be framed with gold as if it were THE problem-solving theory ~~explaining~~ describing gravitation.
Please also read http://henk-reints.nl/astro/HR-Deflection-of-light-passing-a-mass.pdf



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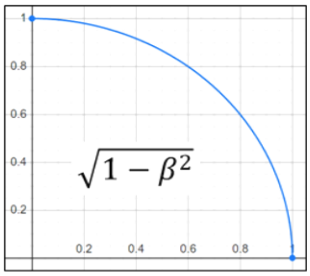

Sophisticated hard way ***yields very same as*** ***easy & comprehensible way.***

In my experience, laymen hardly ever ***understand*** *curved spacetime*
& it's the *recipient* who determines the clarity of a message, not the sender!

Please let them read: http://henk-reints.nl/astro/HR-Relativity-and-curvature-of-spacetime.pdf .

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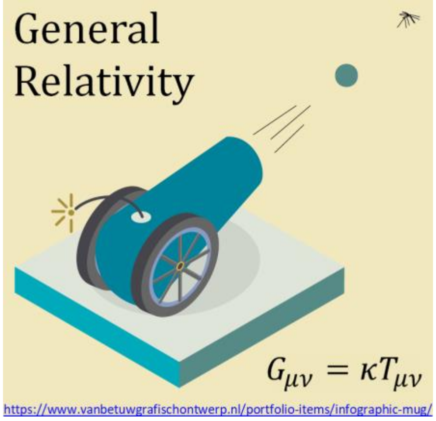
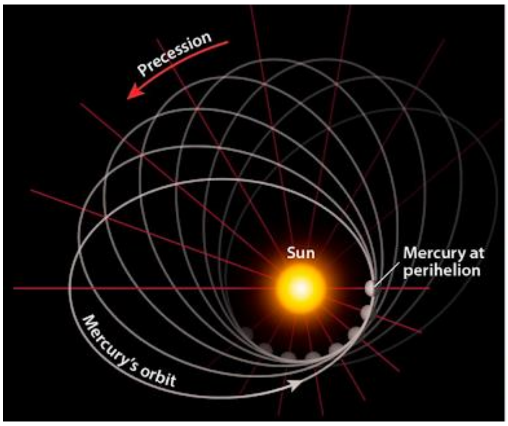
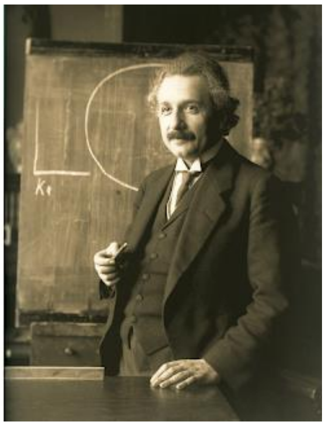
[4] See https://www.aanda.org/articles/aa/full_html/2020/04/aa37813-20/aa37813-20.html
[5] From: "New Perspectives In Physics" by Louis de Broglie (1962); he litterally wrote: "He told me"
https://archive.org/details/newperspectivesi00brog/page/184/mode/1up
https://books.google.nl/books?id=xY45AAAAMAAJ&q=%22mathematical+expression+apart%22&redir_esc=y#search_anchor







Einstein's equation for perihelion precession had already been found in 1898 by Paul Gerber (1854–1909), although he ASSUMED gravity needs time to overcome a distance: Gerber P. (1898) Die räumliche und zeitliche Ausbreitung der Gravitation. Zeits.f.Math.u.Phys. 43, 93-104.
https://archive.org/details/zeitschriftfrma14runggoog/page/n101/mode/2up?view=theater
https://de.wikisource.org/wiki/Die_r%C3%A4umliche_und_zeitliche_Ausbreitung_der_Gravitation

In 1916, a few months after Einstein's publication of GR, Ernst Gehrcke criticised Einstein[6]. In 1920, Einstein wrote[7]: *Mr. Gehrcke wants to make us believe that the perihelion shift of Mercury can be explained without the theory of relativity. So there are two possibilities. Either you invent special interplanetary masses.* (...) *Or you rely on a work by Gerber, who already gave the right formula for the perihelion shift of Mercury before me. The experts are not only in agreement that Gerber's derivation is wrong through and through, but the formula cannot be obtained as a consequence of the main assumption made by Gerber* [HR: Lieber Albert, haben Sie nicht geschrieben: "*Autoritätsdusel ist der größte Feind der Wahrheit*"?]. *Mr. Gerber's work is therefore completely useless, an unsuccessful and erroneous theoretical attempt. I maintain that the theory of general relativity has provided the first real explanation of the perihelion motion of Mercury. I did not mention the work by Gerber initially, because I did not know about it when I wrote my work on the perihelion motion of Mercury; even if I had been aware of it, I would not have had any reason to mention it.*

I consider Einstein's remark rather unqualified; he did not put his finger on any error by Gerber. The experts he mentions must be H. Seeliger[8] & M. von Laue[9]. Gerber's derivation essentially is: **IF** gravity has a velocity, then perihelion shift must exist according to a specific equation containing measurable quantities, from which we can derive this velocity. His equation ***IS*** the correct one and from Mercury's *observed* data, he finds: 305500 km/s. *What is the probability of finding an exactly correct equation via one or more errors?* I see no errors, but that may be a shortage of my own. If you can put your finger on a true error by Gerber, please let me know.

A ***physical*** **explanation** of ~~perihelion~~ orbit shift is possible using not more than special relativity in combination with what I coined *gravocity*, the gravitational pseudo velocity that arises from the *equivalence principle*, plus the ASSUMPTION that the orbital period would be frame independent (which however can be concluded from Einstein's GR-solution, so ultimately, he wins the game). *Gravitational time dilation* should then be regarded as *kinematic time dilation* due to the *gravocity*.

A mathematical notation of the ***Equivalence Principle*** would be:

Schwarzschild's root = Lorentz's: $$\sqrt{1-\frac{2GM}{rc^2}} = \sqrt{1-\frac{v_{\mathrm{ff}}^2}{c^2}}$$

hence: $$v_{\mathrm{ff}}^2 = \frac{2GM}{r} \quad \therefore \frac{1}{2}mv_{\mathrm{ff}}^2 - \frac{GMm}{r} = 0$$

the latter being (Newtonian): $E_{\mathrm{kin}} + E_{\mathrm{pot}} = 0$, i.e. ***conservation of energy.***

Kinetic energy is due to acceleration, potential energy arises from countering gravitation, so there you essentially have it: the energy-based equivalence of acceleration and gravitation.

Henk Reints
Henk-Reints.nl

[6] E. Gehrke: Zur Kritik und Geschichte der neueren Gravitationstheorien. Annalen der Physik. 51 (17): 119–124.
[7] https://en.wikipedia.org/wiki/Paul_Gerber
[8] H.Seeliger: Bemerkungen zu P. Gerbers Aufsatz: "Die Fortpflanzungsgeschwindigkeit der Gravitation". Annalen der Physik. 53 (9): 31–32.
[9] M. von Laue Die Fortpflanzungsgeschwindigkeit der Gravitation. Bemerkungen zur gleichnamigen Abhandlung von P. Gerber. Annalen der Physik. 53 (11): 214–216.

