Scenario: stationary receiver, stationary medium, emittor leaves and returns;

wave velocity: c
velocity of emittor: v $\beta = v/c$
travel distance: r
emitted frequency: f_e

Classical Doppler effect:

way out:

- received frequency: $f_{\mathrm{r},0} = f_\mathrm{e} \frac{c}{c+v}$
- duration of emission: $\Delta t_{\mathrm{e},0} = \frac{r}{v}$
- emitted periods: $n_{\mathrm{e},0} = \Delta t_{\mathrm{e},0} \cdot f_\mathrm{e} = \frac{r f_\mathrm{e}}{v}$
- travel time of last period: $\Delta t_\mathrm{L} = \frac{r}{c}$
- duration of reception: $\Delta t_{\mathrm{r},0} = \Delta t_{\mathrm{e},0} + \Delta t_\mathrm{L} = \frac{r}{v} + \frac{r}{c} = \frac{r(c+v)}{cv}$
- received periods: $n_{\mathrm{r},0} = \Delta t_{\mathrm{r},0} \cdot f_{\mathrm{r},0} = \frac{r(c+v)}{cv} \cdot f_\mathrm{e} \frac{c}{c+v} = \frac{r f_\mathrm{e}}{v}$

way home:

- received frequency: $f_{\mathrm{r},1} = f_\mathrm{e} \frac{c}{c-v}$
- duration of emission: $\Delta t_{\mathrm{e},1} = \frac{r}{v}$
- emitted periods: $n_{\mathrm{e},1} = \Delta t_{\mathrm{e},1} \cdot f_\mathrm{e} = \frac{r f_\mathrm{e}}{v}$
- travel time of first period: $\Delta t_\mathrm{i} = \frac{r}{c}$
- duration of reception: $\Delta t_{\mathrm{r},1} = \Delta t_{\mathrm{e},1} - \Delta t_\mathrm{i} = \frac{r}{v} - \frac{r}{c} = \frac{r(c-v)}{cv}$
- received periods: $n_{\mathrm{r},1} = \Delta t_{\mathrm{r},1} \cdot f_{\mathrm{r},1} = \frac{r(c-v)}{cv} \cdot f_\mathrm{e} \frac{c}{c-v} = \frac{r f_\mathrm{e}}{v}$

please note: $\Delta t_{\mathrm{e},0} = \Delta t_{\mathrm{e},1}$ & $n_{\mathrm{e},0} = n_{\mathrm{e},1}$ & $n_{\mathrm{r},0} = n_{\mathrm{r},1}$

total periods emitted: $n_\mathrm{e} = n_{\mathrm{e},0} + n_{\mathrm{e},1} = \frac{2 r f_\mathrm{e}}{v}$

total periods received: $n_\mathrm{r} = n_{\mathrm{r},0} + n_{\mathrm{r},1} = \frac{2 r f_\mathrm{e}}{v}$

hence: $n_\mathrm{r} = n_\mathrm{e}$ (how trivial...)

Relativistic Doppler effect:

equations that – as observed in the stationary's proper frame – yield the same as above are omitted below;

way out:

received frequency: $f_{\mathrm{r},0} = f_\mathrm{e} \cdot \sqrt{\frac{1-\beta}{1+\beta}}$

received periods:

$$n_{\mathrm{r},0} = \Delta t_{\mathrm{r},0} \cdot f_{\mathrm{r},0} = \frac{r(c+v)}{cv} \cdot f_\mathrm{e}\sqrt{\frac{1-\beta}{1+\beta}} = \frac{rf_\mathrm{e}}{v} \cdot \frac{(c+v)}{c} \cdot \sqrt{\frac{1-\beta}{1+\beta}}$$

$$= \frac{n_\mathrm{e}}{2}(1+\beta) \cdot \sqrt{\frac{1-\beta}{1+\beta}} = \frac{n_\mathrm{e}}{2} \cdot \sqrt{(1+\beta)(1-\beta)}$$

$$= \frac{n_\mathrm{e}}{2} \cdot \sqrt{1-\beta^2}$$

way home:

received frequency: $f_{\mathrm{r},1} = f_\mathrm{e} \cdot \sqrt{\frac{1+\beta}{1-\beta}}$

received periods:

$$n_{\mathrm{r},1} = \Delta t_{\mathrm{r},1} \cdot f_{\mathrm{r},1} = \frac{r(c-v)}{cv} \cdot f_\mathrm{e} \cdot \sqrt{\frac{1+\beta}{1-\beta}} = \frac{rf_\mathrm{e}}{v} \cdot \frac{(c-v)}{c} \cdot \sqrt{\frac{1+\beta}{1-\beta}}$$

$$= \frac{n_\mathrm{e}}{2}(1-\beta) \cdot \sqrt{\frac{1+\beta}{1-\beta}} = \frac{n_\mathrm{e}}{2} \cdot \sqrt{(1-\beta)(1+\beta)}$$

$$= \frac{n_\mathrm{e}}{2} \cdot \sqrt{1-\beta^2}$$

please note: $n_{\mathrm{r},0} = n_{\mathrm{r},1}$

total periods received: $n_\mathrm{r} = n_{\mathrm{r},0} + n_{\mathrm{r},1} = n_\mathrm{e} \cdot \sqrt{1-\beta^2}$

Not all emitted wave periods have been received!

Where did they go?

Time dilation:

MUST have received all: $n_{\mathrm{r,actual}} = n_{\mathrm{e}}$

hence: $n_{\mathrm{r,actual}} = n_{\mathrm{r}} \cdot \frac{1}{\sqrt{1-\beta^2}}$

at the mean reception frequency: $f_{\mathrm{r}} = \frac{n_{\mathrm{r}}}{\Delta t_{\mathrm{r}}}$

the reception duration must
retrospectively have been: $\Delta t_{\mathrm{r,retrospective}} = \frac{n_{\mathrm{r,actual}}}{f_{\mathrm{r}}} = \frac{n_{\mathrm{r}}}{f_{\mathrm{r}}} \cdot \frac{1}{\sqrt{1-\beta^2}} = \Delta t_{\mathrm{r}} \cdot \frac{1}{\sqrt{1-\beta^2}}$

so retrospectively, it must have lasted longer, cf. the muons.

Since both observers are together at the start AND the end of the journey,
we have: $\Delta t_{\mathrm{r}} = \Delta t_{\mathrm{e}}$
so: $\Delta t_{\mathrm{r,retrospective}} = \Delta t_{\mathrm{e}} \cdot \frac{1}{\sqrt{1-\beta^2}}$

The very first period pretends to have been received before it was emitted.

Also: $\Delta t_{\mathrm{r,retrospective}} = \Delta t_{\mathrm{e,retrospective}}$

therefore: $\Delta t_{\mathrm{retrospective}} = \Delta t \cdot \frac{1}{\sqrt{1-\beta^2}}$

The total wave emission retrospectively

lasted longer than how much time it took

by a factor of $\gamma = 1/\sqrt{1-\beta^2}$

That's how time dilation/stretching works.

See also: http://henk-reints.nl/astro/HR-Twin-paradox-slides.pdf

https://thumbs.dreamstime.com/b/low-tide-boats-english-harbour-mousehole-cornwall-england-uk-cornish-fishing-village-blue-sky-clouds-35114227.jpg

Missing waves

