Jeans length: $R_\mathrm{J}^2 = \frac{15 k_\mathrm{B} T_0}{4\pi G m \rho_0}$

Jeans mass: $M_\mathrm{J} = \frac{4\pi}{3} R_\mathrm{J}^3 \cdot \rho_0$

hence: $\frac{M_\mathrm{J}}{R_\mathrm{J}} = \frac{4\pi\rho_0}{3} R_\mathrm{J}^2 = \frac{4\pi\rho_0}{3} \cdot \frac{15 k_\mathrm{B} T_0}{4\pi G m \rho_0} = \frac{5 k_\mathrm{B} T_0}{G m}$

also: $N = \frac{M_\mathrm{J}}{m}$

initial potential energy: $E_{\mathrm{p},0} = \frac{-3 G M_\mathrm{J}^2}{5 R_\mathrm{J}}$

The virial theorem applies to systems in equilibrium. A collapse results from the absence of an equilibrium, so the virial theorem does not apply to the final kinetic energy. Moreover, we are interested in the maximum possible resulting temperature, so we convert *all* initial potential energy to kinetic:

post-collapse kinetic energy: $E_\mathrm{k} = N \cdot \frac{3}{2} k_\mathrm{B} T \leq -E_{\mathrm{p},0} = \frac{3 G M_\mathrm{J}^2}{5 R_\mathrm{J}}$

or: $\frac{M_\mathrm{J}}{m} \cdot k_\mathrm{B} T \leq \frac{2 G M_\mathrm{J}^2}{5 R_\mathrm{J}}$

hence: $T \leq \frac{2 G m}{5 k_\mathrm{B}} \cdot \frac{M_\mathrm{J}}{R_\mathrm{J}} = \frac{2 G m}{5 k_\mathrm{B}} \cdot \frac{5 k_\mathrm{B} T_0}{G m} = 2 T_0$

A gravitational collapse would yield
at most twice the initial temperature!

Adiabatic compression occurs when a gas is compressed against its own pressure. During a collapse, such a counter pressure is not available, so adiabatics equations do not apply. No more work can be done than what is given by the initial potential energy.

Assuming the pre-collapse cloud was in near equilibrium, the virial theorem says there already was an initial amount of kinetic energy equal to $-E_{\mathrm{p},0}/2$, which must yet be added to the post-collapse result, yielding:

$$T \leq 3 T_0 \,.$$

However, this is the average of the entire cloud. At its centre it will be compressed far more that near its edge. Energy is transferred from the outer regions to the centre, so the central temperature can easily be significantly higher, at the cost of way colder outer regions of the cloud. If the initial potential energy would become concentrated as kinetic energy of, for example, just the central 1% of the mass, the cloud's central temperature would become 100 times higher, yielding $100 \times 3 \times 10$ K $= 3000$ K.

This approximates the observed temperature of protostars.

