Transverse Hubble velocity?

Whilst watching some children in a playground, I was contemplating that in a carousel the velocity is proportional to the distance to its centre and all of a sudden Hubble's law flashed through my head: velocity proportional to the distance. Could it be that the Hubble constant actually is an angular velocity? The entire universe simply rotating around its very centre, the latter then obviously being the earth, urh, no, me myself? It thawed and I taught myself it was a distorted taut thought. But why not do some calculations? Although I think the idea is in conflict with the Cosmological Principle, we can at least give it a try, can't we?

First, we define the redshift factor (Doppler factor): $\zeta \equiv z + 1.$

A celestial object at a distance: d
has a redshift: z
that, using the longitudinal relativistic Doppler effect,
can be reduced to a longitudinal velocity: $v_l = c \cdot \frac{\zeta^2-1}{\zeta^2+1}$
which follows the Hubble-Lemaître law: $v_l = H_0 \cdot d$
and then the distance equals: $d = \frac{v_l}{H_0} = \frac{c}{H_0} \cdot \frac{\zeta^2-1}{\zeta^2+1}.$

Now suppose this redshift actually is due to a transverse velocity only,
whilst the just derived distance would still be the true distance.

Then the transverse relativistic Doppler effect would apply, yielding: $v_t = c \cdot \frac{\sqrt{\zeta^2-1}}{\zeta}$

rendering an angular velocity of: $\omega = \frac{v_t}{d} = \frac{c \cdot \frac{\sqrt{\zeta^2-1}}{\zeta}}{\frac{c}{H_0} \cdot \frac{\zeta^2-1}{\zeta^2+1}} = H_0 \cdot \frac{\sqrt{\zeta^2-1}}{\zeta} \cdot \frac{\zeta^2+1}{\zeta^2-1} = H_0 \cdot \frac{\zeta^2+1}{\zeta\sqrt{\zeta^2-1}}$ rad/s (with H_0 in s^{-1}).

We should however have used the distance at which the now observed light was emitted, because that is how we actually see the object. In http://henk-reints.nl/astro/HR-on-the-universe.php this emission distance is derived as

equation [53]: $\rho_e \left(= \frac{d_e}{d_H}\right) = \frac{\beta}{1+\beta} = \frac{\frac{\zeta^2-1}{\zeta^2+1}}{1+\frac{\zeta^2-1}{\zeta^2+1}} = \frac{\zeta^2-1}{2\zeta^2}$ $\quad (d_H \equiv \frac{c}{H_0}$ is the Hubble distance)

and then: $d_e = \frac{c}{H_0} \cdot \frac{\zeta^2-1}{2\zeta^2}$

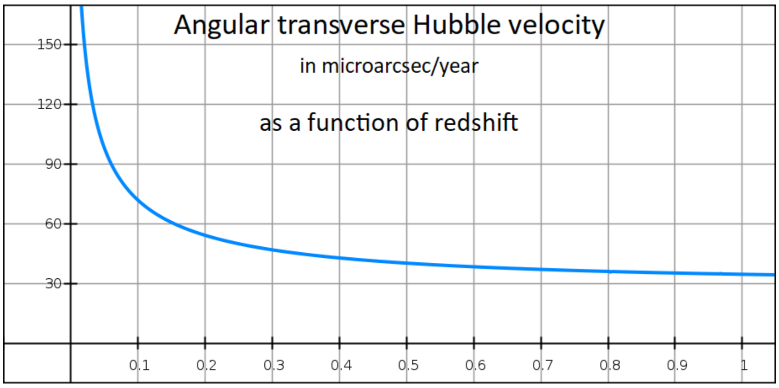
yielding: $\omega = \frac{v_t}{d_e} = \frac{c \cdot \frac{\sqrt{\zeta^2-1}}{\zeta}}{\frac{c}{H_0} \cdot \frac{\zeta^2-1}{2\zeta^2}} = H_0 \cdot \frac{\sqrt{\zeta^2-1}}{\zeta} \cdot \frac{2\zeta^2}{\zeta^2-1} = 2H_0 \cdot \frac{\zeta}{\sqrt{\zeta^2-1}}$ rad/s.

This equals: $\omega = \frac{180 \times 60 \times 60}{\pi} \cdot 2H_0 \cdot \frac{\zeta}{\sqrt{\zeta^2-1}}$ arcsec/s

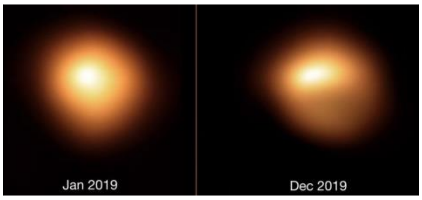
or: $\omega = 365.25 \times 24 \times 60 \times 60 \cdot \frac{180 \times 60 \times 60}{\pi} \cdot 2H_0 \cdot \frac{\zeta}{\sqrt{\zeta^2-1}}$ arcsec/year.

With: $H_0 = 71$ km/s/Mpc $= 2.3 \times 10^{-18}$ /s

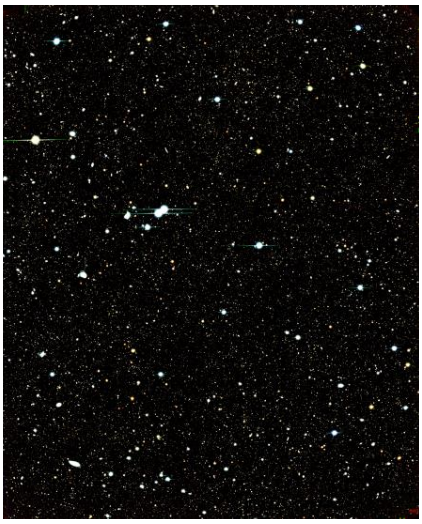
we then obtain: $\omega \approx 30 \cdot \frac{\zeta}{\sqrt{\zeta^2-1}} = 30 \cdot \frac{z+1}{\sqrt{z^2+2z}}$ microarcsec/year.



If it would be possible at all to directly measure such a small angular velocity of at most say 150 microarcsec/year, it would apply only to nearby objects. What would it say about the entire universe?



As a bad example as far as Hubble movement is concerned, the well-known star Betelgeuse in the constellation of Orion (upper left as seen from the north) has a radius of $887 \pm 203\,\mathrm{R}_\odot$ and resides at a distance of approximately $700\,\mathrm{ly}$, yielding an apparent diameter of ~ 38 milliarcsec $= 38\,000$ microarcsec. That is what the image to the right tells us about the resolution we have achieved up to now. Let's say it is about a quarter of this apparent diameter, or do you see more details?



On http://soaps.nao.ac.jp/SDF/v1/sdf_v1_README we find that the Subaru Deep Field image (revealing 1.4 mln. distant objects) spans 0.202 arc seconds per pixel in the original 8900×11000 image, so its unsharpness is $\sim 200\,000$ microarcsec, which is too large by a factor of well over 1000.

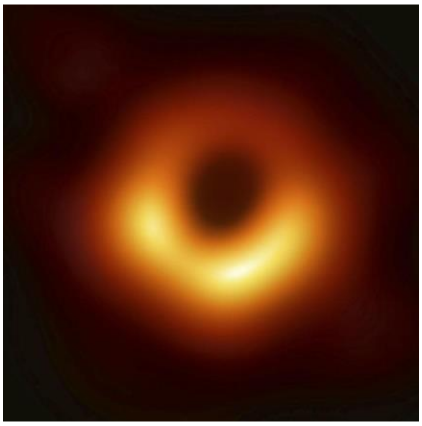
https://www.nasa.gov/missions/highlights/webcasts/shuttle/sts109/hubble-qa.html gives the unsharpness (usually called resolution, but that is in fact an incorrect name) of the Hubble Space Telescope as 0.04 arc seconds $= 40\,000$ microarcsec. Too large by a factor of over 250.

Let's have a look at a large telescope. First of all we consider the radius of an Airy disk: $\theta = 1.22 \cdot \frac{\lambda}{d}$ radians $= 251643 \times 10^6 \cdot \frac{\lambda}{d}$ microarcsec, where λ is the wavelength and d is the aperture of the lens. Both must of course be expressed in the same unit.

The Keck 1 & 2 telescopes on Mauna Kea, Hawaii have an aperture of 10 metres and the wavelength of visible light is circa 500 nanometres, yielding an unsharpness of:

$$\theta = 251643 \times 10^6 \cdot \frac{500 \times 10^{-9}}{10} = 12\,582 \text{ microarcsec}$$

and there we have the aforementioned quarter of Betelgeuse's apparent diameter. It is insufficient by a factor of well over 100.



And then there is this black hole image, where the effective aperture of the telescope array (some 10 telescope were combined to produce the image) approaches the size of the earth. According to https://en.wikipedia.org/wiki/Messier_87

the thing has a mass of: $6.5 \times 10^9\, M_\odot = 1.3 \times 10^{40}$ kg,
yielding a Schwarzschild radius of: $r_S = 1.9 \times 10^{13}$ metres.
It resides at a distance of: $d = 53.5 \times 10^6$ light years
$= 5.1 \times 10^{23}$ metres,
yielding an angular diameter of: $\frac{2 r_S}{d} = 15.7$ microarcsec.

The shadow of the black hole (the central dark area in the image to the right) is 2.6 times as large, i.e. 41 microarcsec in diameter.

Therefore I conclude that a global array of (radio) telescopes would be able to do the job. On https://www.cfa.harvard.edu/news/2005-07 I read that the transverse motion of M33 (at only 2.4 mln. light years) has been measured in 2005 as 30 microarcsec/year, far less than the aforementioned 150, so it might falsify the silly assumption of transverse Hubble motion. But is has a sample size of just one.

So falsification with a 5σ reliability is still a long way to go. But, as said, I think it is in conflict with the Cosmological Principle, as is stated on https://iopscience.iop.org/article/10.3847/1538-4357/aad3d0.

There are 10 types of people. Those who understand binary and those who don't.