Andromeda & Milky Way galaxies in free fall towards each other

IF initial mutual velocity = 0 m/s (both radial and orbital velocity).

(Andromeda actually has a radial velocity towards us of rougly 300 km/s, but we hardly know its lateral velocity, so it might well be that we are revolving our common barycentre).

We consider the resulting bull's eye trajectory to be a degenerate elliptical orbit $[a < \infty,\ b = 0 \Rightarrow e = 1]$ from apoapsis to periapsis according to Kepler's laws, where the reduced mass $\mu = Mm/(M+m)$ "orbits" the $M+m$ barycentre and we calculate half the orbital period.

$$\text{Kepler's 3}^{\text{rd}}\text{ law:}\quad \omega^2 a^3 = GM_{\text{tot}} \therefore \omega = \sqrt{\frac{GM_{\text{tot}}}{a^3}}$$

$$\text{we have:}\quad a = \frac{R}{2} \therefore a^3 = \frac{R^3}{2^2 2}$$

$$\omega = \frac{2\pi}{T} \therefore T = \frac{2\pi}{\omega} = 2\pi \cdot \sqrt{\frac{a^3}{GM_{\text{tot}}}} = \frac{2\pi}{2} \cdot \sqrt{\frac{R^3}{2GM_{\text{tot}}}}$$

$$\text{hence, the } \textit{free fall time} \text{ (½orbit) is:}\quad \boldsymbol{t_{\text{ff}}} = \frac{T}{2} = \boldsymbol{\frac{\pi}{2} \cdot \sqrt{\frac{R^3}{2GM_{\text{tot}}}}}$$

<u>We have:</u>

$R \approx 2.54 \times 10^6$ ly $\approx 2.403 \times 10^{22}$ m

$G \approx 6.67430 \times 10^{-11}$ N·m²/kg²

$M_{\text{MW}} \approx 1.15 \times 10^{12} M_\odot$

$M_{\text{Andr}} \approx 1.5 \times 10^{12} M_\odot$

$M_\odot \approx 1.98847 \times 10^{30}$ kg

$M_{\text{tot}} = M_{\text{MW}} + M_{\text{Andr}} \approx 5.27 \times 10^{42}$ kg

<u>yielding:</u>

$$\boldsymbol{t_{\text{ff}} \approx 2.21 \times 10^{17}\ \text{s} \approx 6.99 \times 10^{9}\ \text{yr} \approx 0.508 t_{\text{H}}}$$

At the aforementioned speed of 300 km/s (wrongly assuming it constant), it would take 2.403×10^{22} m / 300 km/s ≈ 2.54×10^9 yr ≈ $0.184 t_{\text{H}}$.

Andromeda is our nearest neighbour spiral galaxy.
On the cosmic scale, it is *very* close to us
and yet our kiss (starting with zero velocity) would have to bide

seven billion years, half the age of the universe!

Intergalactic gravitation is

GIGANTICALLY small.

Yes, it's astronomically small, negligible for most purposes.

We see this fleet need no beat; it neatly meets & the speed be:

(of course it's a one-dimensional head-on collision and we calculate in the barycentric frame!)

conservation of energy: $E_{\text{kin,tot}} = \frac{1}{2} M_A v_A^2 + \frac{1}{2} M_{MW} v_{MW}^2 = \frac{G M_A M_{MW}}{R} = E_{\text{pot}}$

conservation of momentum: $\frac{v_A}{v_{MW}} = \frac{M_{MW}}{M_A}$ (using absolute values)

hence: $v_{MW} = v_A \frac{M_A}{M_{MW}} \therefore v_{MW}^2 = v_A^2 \frac{M_A^2}{M_{MW}^2}$

therefore: $\frac{1}{2} M_A v_A^2 + \frac{1}{2} M_{MW} v_A^2 \frac{M_A^2}{M_{MW}^2} = \frac{G M_A M_{MW}}{R}$

i.e.: $v_A^2 M_A \left(1 + \frac{M_A}{M_{MW}}\right) = \frac{2 G M_A M_{MW}}{R}$

yielding: $v_A = \sqrt{\frac{2 G M_A M_{MW}}{R M_A \left(1 + \frac{M_A}{M_{MW}}\right)}} = \sqrt{\frac{2 G M_{MW}^2}{R (M_A + M_{MW})}} = M_{MW} \sqrt{\frac{2G}{R (M_A + M_{MW})}}$

and: $v_{MW} = v_A \frac{M_A}{M_{MW}} = M_A \sqrt{\frac{2G}{R (M_A + M_{MW})}}$

which results in: $\boldsymbol{v}_{\textbf{coll}} = (M_A + M_{MW}) \sqrt{\frac{2G}{R (M_A + M_{MW})}} = \sqrt{\frac{\boldsymbol{2G(M_A + M_{MW})}}{\boldsymbol{R}}}$

This renders: **~171 km/s**

Cf.: ~200 km/s < $v_{\odot,MW}$ < ~250 km/s

Dividing the sum of their radii by this velocity
yields the **maximum duration of this collision**:

we find:	R_{MW}	≈ 52850 ly	(Google as of 2024-03-26)
and:	R_A	≈ 110000 ly	(sic)
so:	R_{tot}	≈ 162850 ly ≈ 1.54×10^{21} m	

yielding: $\boldsymbol{\Delta t_{\text{coll}}}$ **< ~285 _m_illion years.**

How many stars will collide? See http://henk-reints.nl/astro/HR-Galaxy-star-collision.pdf.

Now assume the sun's orbital period around the galactic centre (~240 Ma) equals the rotation period of the entire **Milky Way**. Then the latter has **revolved** not more than a mere 13.77 Ga / 240 Ma ≈ **57 times since the big bang.**

Ceci n'est pas la galaxie d'Andromède, ni la Voie lactée.