Note: many numerical results below are in full precision as given by javascript or Google, which in fact is quite useless...

We will start with some info on the Golden Ratio:

It is defined as the ratio such that: $total : largest \ = \ largest : smallest$

or, as a formula: $(a+b) : a = a : b$

its value $\varphi \equiv a/b$ equals: $\varphi = \frac{1+\sqrt{5}}{2} \approx 1.618\,033\,988\,75$

some interesting properties:

$\varphi^2 = 1 + \varphi$ $\quad\therefore \varphi = \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\cdots}}}}$

$\Phi \equiv \frac{1}{\varphi} = \varphi - 1$ $\quad\therefore \varphi = 1 + \frac{1}{\varphi}$ $\quad\therefore \varphi = 1 + \frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{1+\cdots}}}}$

$4/\sqrt{\varphi} \approx \pi$ (a mathematical coincidence within $0.96‰$)

In <u>ANY</u> sequence where the next number equals the sum of the prior two, no matter what the starting numbers are (unless both are zero), the ratio of succeeding numbers approaches the Golden Ratio as the sequence grows. Most well-known is the sequence of Fibonacci ("figlio di Bonacci" = Bonacci's son, his actual name was Leonardo Pisano, Leonard of Pisa), which starts with $1, 1$. It is however just one (and maybe the simplest) of such sequences.

Now suppose a golden pyramid, i.e. both its perpendicular midsections (not its diagonal sections) are a pair of golden triangles (Kepler triangles). Such a triangle's *sides*

are in the ratio of: $1 : \sqrt{\varphi} : \varphi$

and it is a rectangular triangle: $1^2 + \sqrt{\varphi}^2 = \varphi^2$ (i.e. $1 + \varphi = \varphi^2$) (Πυθαγορας)

Let the *base* of each triangle be: $b_g = 1$

then its *height* equals: $h_g = b_g\sqrt{\varphi}$

and its *hypotenuse* is: $y_g = b_g\varphi$

This pyramid's *side* would be: $s_g = 2b_g = 2$

and its *circumference* equals: $p_g = 4s_g = 8$

Also suppose a pyramid which has a *circumference* of 2π times its *height*, as if the equator of a hemisphere with a *radius* equal to that *height* has been reshaped to have a square base with the same *circumference*. A quadratic hemisphere so to say. Let's call it a "quamispheramid".

It has a *height* of: $h_q = h_g$

which we simply call: $h = \sqrt{\varphi}$

then is *circumference* is: $p_q = 2\pi\sqrt{\varphi}$ $\approx 7.992\,335\,172\,270\,526$ (cf. p_g)

its *side* equals: $s_q = \frac{p_q}{4}$ $\approx 1.998\,083\,793\,067\,631\,4$ (cf. s_g)

and half thereof is: $b_q = \frac{s_q}{2} = \frac{\pi h}{4}$ $\approx 0.999\,041\,896\,533\,815\,7$ (cf. b_g)

For the ratio of these *circumferences*

we find: $\frac{p_g}{p_q} \approx 1.000\,959\,022\,308\,782\,5$

and: $\frac{p_q}{p_g} \approx 0.999\,041\,896\,533\,815\,7$

(it's not my fault that q and g are look-alikes, but one might blame me for using them... ☺).

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As should be obvious, the quamispheramid fits just inside the golden pyramid whilst they share their very tops.

This makes us consider a single building that just looks like a four-sided pyramid, but it actually is an octagonal combination of the above. The corners of its base would be those of a golden pyramid and the midpoints of each side would be those of a quamispheramid, so all four sides of the building's base line would be indentend towards the centre. For this *indentation*

we find:
$$i = b_g - b_q = b_g - \frac{\pi h}{4} = b_g - \frac{\pi b_g \sqrt{\varphi}}{4} \qquad (i \neq \sqrt{-1})$$
$$= b_g\left(1 - \frac{\pi}{4}\sqrt{\varphi}\right) = s_g\left(\frac{1}{2} - \frac{\pi}{8}\sqrt{\varphi}\right)$$
$$\approx b_g \cdot 0.000\,958\,103\,466\,184\,301\,3$$
or:
$$\approx s_g \cdot 0.000\,479\,051\,733\,092\,150\,65$$

Let's denote the corner-to-corner *distance* with a simple s,

then:
$$i = s\left(\frac{1}{2} - \frac{\pi}{8}\sqrt{\varphi}\right)$$
or:
$$i \approx 0.48‰ \text{ of the corner-to-corner } \textit{distance}.$$

For a corner-to-corner *distance* of 100 metres this would be 4.8 centimetres.

For the *distance* from a corner to the

indented midpoint of a *side* we find:
$$l = \sqrt{b_g^2 + i^2} \approx b_g \cdot 1.000\,000\,458\,981\,020\,7 \qquad (i^2 \neq -1)$$

For the *surface area* of a face of the building we will do an extensive step by step OMG derivation. You are allowed to skip it and directly jump to the result. A triangle's area of course is: $A_\triangle = \frac{bh}{2}$, so:

$$A = 2 \cdot \frac{l \cdot \sqrt{b_q^2 + h^2}}{2}$$ which of course is twice the corner-midpoint-top triangle.

$$= \sqrt{l^2\left(b_q^2 + h^2\right)} = \sqrt{\left(b_g^2 + i^2\right)\left(b_q^2 + h^2\right)} = \sqrt{\left\{b_g^2 + \left(b_g - b_q\right)^2\right\}\left(b_q^2 + h^2\right)}$$
$$= \sqrt{\left\{b_g^2 + \left(b_g - \frac{\pi h}{4}\right)^2\right\}\left\{\left(\frac{\pi h}{4}\right)^2 + h^2\right\}} = \sqrt{\left\{b_g^2 + \left(b_g - \frac{\pi b_g\sqrt{\varphi}}{4}\right)^2\right\}\left\{\left(\frac{\pi b_g\sqrt{\varphi}}{4}\right)^2 + \left(b_g\sqrt{\varphi}\right)^2\right\}}$$
$$= \sqrt{\left\{b_g^2 + \left(b_g^2 - 2b_g\frac{\pi b_g\sqrt{\varphi}}{4} + \frac{\pi^2 b_g^2\varphi}{16}\right)\right\}\left(\frac{\pi^2 b_g^2\varphi}{16} + b_g^2\varphi\right)} = \sqrt{\left(b_g^2 + b_g^2 - \frac{\pi b_g^2\sqrt{\varphi}}{2} + \frac{\pi^2 b_g^2\varphi}{16}\right)\left(\frac{\pi^2 b_g^2\varphi}{16} + b_g^2\varphi\right)}$$
$$= \sqrt{b_g^2\left(2 - \frac{\pi\sqrt{\varphi}}{2} + \frac{\pi^2\varphi}{16}\right)b_g^2\left(\frac{\pi^2\varphi}{16} + \varphi\right)} = b_g^2 \cdot \sqrt{\left(2 - \frac{\pi\sqrt{\varphi}}{2} + \frac{\pi^2\varphi}{16}\right)\left(\frac{\pi^2\varphi}{16} + \varphi\right)}$$
$$= \frac{h^2}{\varphi} \cdot \sqrt{\left(2 - \frac{\pi\sqrt{\varphi}}{2} + \frac{\pi^2\varphi}{16}\right)\left(\frac{\pi^2\varphi}{16} + \varphi\right)} = \frac{h^2}{\varphi} \cdot \sqrt{\frac{\pi^2\varphi}{16}\left(2 - \frac{\pi\sqrt{\varphi}}{2} + \frac{\pi^2\varphi}{16}\right) + \varphi\left(2 - \frac{\pi\sqrt{\varphi}}{2} + \frac{\pi^2\varphi}{16}\right)}$$
$$= \frac{h^2}{\varphi} \cdot \sqrt{2\frac{\pi^2\varphi}{16} - \frac{\pi\sqrt{\varphi}}{2} \cdot \frac{\pi^2\varphi}{16} + \frac{\pi^2\varphi}{16} \cdot \frac{\pi^2\varphi}{16} + 2\varphi - \frac{\pi\sqrt{\varphi}}{2}\varphi + \frac{\pi^2\varphi}{16}\varphi}$$
$$= \frac{h^2}{\varphi} \cdot \sqrt{\frac{\pi^2\varphi}{8} - \frac{\pi^3\varphi\sqrt{\varphi}}{32} + \frac{\pi^4\varphi^2}{256} + 2\varphi - \frac{\pi\varphi\sqrt{\varphi}}{2} + \frac{\pi^2\varphi^2}{16}} = \frac{h^2}{\varphi} \cdot \sqrt{\frac{\pi^2\varphi^2}{16} + \frac{\pi^4\varphi^2}{256} + \frac{\pi^2\varphi}{8} + 2\varphi - \frac{\pi^3\varphi\sqrt{\varphi}}{32} - \frac{\pi\varphi\sqrt{\varphi}}{2}}$$
$$= \frac{h^2}{\varphi} \cdot \sqrt{\frac{\pi^2}{16}\left(1 + \frac{\pi^2}{16}\right)\varphi^2 + \left(2 + \frac{\pi^2}{8}\right)\varphi - \pi\left(\frac{1}{2} + \frac{\pi^2}{32}\right)\varphi\sqrt{\varphi}} = h^2 \cdot \sqrt{\frac{\pi^2}{16}\left(1 + \frac{\pi^2}{16}\right) + \frac{2 + \frac{\pi^2}{8}}{\varphi} - \pi\frac{\left(\frac{1}{2} + \frac{\pi^2}{32}\right)}{\sqrt{\varphi}}}$$
$$= h^2 \cdot \sqrt{\frac{\pi^2}{16}\left(1 + \frac{\pi^2}{16}\right) + \frac{2 + \frac{\pi^2}{8}}{\varphi} - \frac{\pi}{4}\frac{\left(2 + \frac{\pi^2}{8}\right)}{\sqrt{\varphi}}} = h^2 \cdot \sqrt{\frac{\pi^2}{16}\left(1 + \frac{\pi^2}{16}\right) + \left(2 + \frac{\pi^2}{8}\right)\left(\frac{1}{\varphi} - \frac{\pi}{4 \cdot \sqrt{\varphi}}\right)}$$
$$= h^2 \cdot \sqrt{\frac{\pi^2}{16}\left(1 + \frac{\pi^2}{16}\right) + \left(2 + \frac{\pi^2}{8}\right)\left(\frac{1}{\varphi} - \frac{\pi \cdot \sqrt{\varphi}}{4\varphi}\right)} = h^2 \cdot \sqrt{\frac{\pi^2}{16}\left(1 + \frac{\pi^2}{16}\right) + \frac{1}{\varphi}\left(2 + \frac{\pi^2}{8}\right)\left(1 - \frac{\pi}{4}\sqrt{\varphi}\right)}$$

which finally results in :
$$A \approx h^2 \cdot 0.999\,634\,604\,244\,164\,1 \approx h^2$$

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For a perfect golden pyramid we have:

height: $h \equiv h_g = b_g\sqrt{\varphi}$

base (*half-side*): $b \equiv b_g = \frac{h}{\sqrt{\varphi}}$

mid-base to top: $y \equiv y_g = b_g\varphi = b\varphi$

surface area of a face: $A = b \cdot y = b^2\varphi = \left(\frac{h}{\sqrt{\varphi}}\right)^2 \varphi$

so: $A_g = h^2$

For a perfect quamispheramid we get:

height: $h \equiv h_q = h_g$

base (*half-side*): $b \equiv b_q = \frac{\pi h}{4}$

mid-base to top: $y = \sqrt{b^2 + h^2}$

surface area of a face: $A = b \cdot y = b \cdot \sqrt{b^2 + h^2} = \frac{\pi h}{4}\sqrt{\frac{\pi^2 h^2}{16} + h^2}$

$= \frac{\pi h}{4}\sqrt{h^2\left(\frac{\pi^2}{16} + 1\right)} = h^2 \cdot \frac{\pi}{4}\sqrt{1 + \frac{\pi^2}{16}}$

so: $A_q \approx h^2 \cdot 0.998\,676\,392\,491\,41$

As should be obvious, the differences between a golden pyramid and a quamispheramid are negligible for practical purposes, unless one explicitly wants to differentiate between the two.

In general, for a square right pyramid (as defined in mathematics) we find next set of formulas. We'll call its *height* and *side* (corner to corner *distance*) h and s, and its perimeter p.

The *ratio*: $\Gamma \equiv \frac{p}{h} = \frac{4s}{h}$ $\therefore h = \frac{4s}{\Gamma}$ and $s = \frac{\Gamma h}{4}$

fully defines the relative geometry.

Slant height (mid-side to apex): $y = \sqrt{\left(\frac{s}{2}\right)^2 + h^2} = \frac{1}{2} \cdot \sqrt{s^2 + 4h^2}$

$= \frac{1}{2} \cdot \sqrt{\left(\frac{\Gamma h}{4}\right)^2 + 4h^2} = \frac{h}{2} \cdot \sqrt{\frac{\Gamma^2}{16} + 4} = h\frac{\sqrt{\Gamma^2+64}}{8}$

$= \frac{4s}{\Gamma} \cdot \frac{\sqrt{\Gamma^2+64}}{8} = s \cdot \frac{\sqrt{\Gamma^2+64}}{2\Gamma}$

$\frac{y}{h} = \frac{\sqrt{\Gamma^2+64}}{8}$

$\frac{y}{s} = \frac{\sqrt{\Gamma^2+64}}{2\Gamma}$

and its *slope* equals: $\alpha = \arctan\frac{h}{s/2} = \arctan\frac{2h}{s} = \arctan\frac{2\left(\frac{4s}{\Gamma}\right)}{s} = \arctan\frac{8}{\Gamma}$

The *corner angle* will be: $\beta = \arctan\frac{y}{s/2} = \arctan\frac{2y}{s} = \arctan\frac{\sqrt{\Gamma^2+64}}{\Gamma}$

The face's *surface area* is: $A_{face} = \frac{sy}{2} = \frac{s}{2} \cdot s \cdot \frac{\sqrt{\Gamma^2+64}}{2\Gamma} = s^2 \cdot \frac{\sqrt{\Gamma^2+64}}{4\Gamma}$

$= \left(\frac{\Gamma h}{4}\right)^2 \frac{\sqrt{\Gamma^2+64}}{4\Gamma} = h^2\frac{\Gamma^2}{16}\frac{\sqrt{\Gamma^2+64}}{4\Gamma} = h^2 \cdot \frac{\Gamma\sqrt{\Gamma^2+64}}{64}$

$\frac{A_{face}}{h^2} = \frac{\Gamma\sqrt{\Gamma^2+64}}{64}$

$\frac{A_{face}}{s^2} = \frac{\sqrt{\Gamma^2+64}}{4\Gamma}$

The ridge will have a *length* of: $r = \sqrt{\left(\frac{s}{2}\right)^2 + y^2} = \sqrt{\left(\frac{s}{2}\right)^2 + \left(\frac{s}{2}\right)^2 + h^2} = \frac{1}{4}\sqrt{4h^2 + 2s^2}$

$= \frac{1}{4}\sqrt{4h^2 + 2\left(\frac{\Gamma h}{4}\right)^2} = \frac{h}{4}\sqrt{4 + 2\frac{\Gamma^2}{16}} = \frac{h}{4}\sqrt{4 + \frac{\Gamma^2}{8}} = \frac{h}{16}\sqrt{64 + 2\Gamma^2}$

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$$= \frac{1}{4}\sqrt{4\left(\frac{4s}{\Gamma}\right)^2 + 2s^2} = \frac{s}{4}\sqrt{\frac{64}{\Gamma^2} + 2}$$

$$\frac{r}{h} = \frac{\sqrt{2\Gamma^2+64}}{16}$$

$$\frac{r}{s} = \frac{1}{4} \cdot \sqrt{2 + \frac{64}{\Gamma^2}}$$

The floor's diagonal is: $d = s\sqrt{2}$

so the ridge's *slope* equals: $\gamma = \arctan\frac{h}{d/2} = \arctan\frac{2h}{s\sqrt{2}} = \arctan\frac{h\sqrt{2}}{s}$

$$= \arctan\frac{\left(\frac{4s}{\Gamma}\right)\sqrt{2}}{s} = \arctan\frac{4\sqrt{2}}{\Gamma}$$

Summarising, without derivations:

perimeter over *height*: $\Gamma \equiv \frac{p}{h} = \frac{4s}{h}$

slant height: $y = \frac{1}{2} \cdot \sqrt{s^2 + 4h^2} = h\frac{\sqrt{\Gamma^2+64}}{8} = s\frac{\sqrt{\Gamma^2+64}}{2\Gamma}$

slant slope: $\alpha = \arctan\frac{2h}{s} = \arctan\frac{8}{\Gamma}$

corner angle: $\beta = \arctan\frac{2y}{s} = \arctan\frac{\sqrt{\Gamma^2+64}}{\Gamma}$

surface area of a single face: $A_{face} = \frac{s^2}{4}\sqrt{1 + \frac{4h^2}{s^2}} = h^2 \cdot \frac{\Gamma\sqrt{\Gamma^2+64}}{64} = s^2 \cdot \frac{\sqrt{\Gamma^2+64}}{4\Gamma}$

ridge length: $r = \frac{1}{4}\sqrt{4h^2 + 2s^2} = \frac{h}{16}\sqrt{2\Gamma^2 + 64} = \frac{s}{4}\sqrt{2 + \frac{64}{\Gamma^2}}$

ridge slope: $\gamma = \arctan\frac{h\sqrt{2}}{s} = \arctan\frac{4\sqrt{2}}{\Gamma}$

A square right pyramid is fully defined by its *side* and *height*, all other values like the *angles* and face *area* are trivial mathematical consequences. Some special pyramids are:

<u>Golden pyramid:</u>

Γ	$= \frac{8}{\sqrt{\varphi}} (\approx 2\pi)$	$\approx 6.289\ 211\ 022\ 06$
h	$= \frac{s}{2}\sqrt{\varphi}$	$\approx s \cdot 0.636\ 009\ 824\ 75$
s	$= \frac{2h}{\sqrt{\varphi}} \left(\approx \frac{\pi h}{2}\right)$	$\approx h \cdot 1.572\ 302\ 755\ 51$
A_{floor}	$= s^2 = \frac{4h^2}{\varphi}$	$\approx h^2 \cdot 2.472\ 135\ 955$
$A_{face} = \frac{s^2}{4}\sqrt{1 + \frac{4h^2}{s^2}} = \frac{s^2}{4} \cdot \sqrt{1+\varphi} = \frac{4h^2}{4\sqrt{\varphi}^2} \cdot \varphi$	$= \frac{\varphi s^2}{4} = h^2$	$\approx s^2 \cdot 0.404\ 508\ 497\ 18$
$y = \frac{1}{2} \cdot \sqrt{s^2 + 4h^2} = \frac{s}{2} \cdot \sqrt{1+\varphi}$	$= \frac{\varphi s}{2} = h \cdot \sqrt{\varphi}$	$\approx s \cdot 0.809\ 016\ 994\ 37$
		$\approx h \cdot 1.272\ 019\ 649\ 51$
$r = \sqrt{h^2 + \frac{1}{2}s^2} = \sqrt{\frac{s^2}{4}\varphi + \frac{2}{4}s^2}$	$= \frac{s}{2} \cdot \sqrt{2+\varphi}$	$\approx s \cdot 0.951\ 056\ 516\ 29$
	$= h \cdot \sqrt{\frac{2+\varphi}{\varphi}} = h \cdot \sqrt[4]{5}$	$\approx h \cdot 1.495\ 348\ 781\ 22$
$\alpha = \arctan\frac{2h}{s}$	$= \arctan\sqrt{\varphi}$	$\approx 51.827\ 292\ 4°$
$\beta = \arctan\sqrt{1 + \frac{4h^2}{s^2}} = \arctan\sqrt{1+\varphi}$	$= \arctan\varphi$	$\approx 58.282\ 526\ 6°$
$\gamma = \arctan\frac{h\sqrt{2}}{s} = \arctan\frac{\sqrt{\varphi}\sqrt{2}}{2}$	$= \arctan\sqrt{\varphi/2}$	$\approx 41.969\ 915\ 2°$

<u>Quamispheramid:</u>

Γ	$= 2\pi \left(\approx \frac{8}{\sqrt{\varphi}}\right)$	$\approx 6.283\ 185\ 307\ 18$

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h	$= \frac{2s}{\pi}$	$\approx s \cdot 0.636\,619\,772\,36$
s	$= \frac{\pi h}{2}\left(\approx \frac{2h}{\sqrt{\varphi}}\right)$	$\approx h \cdot 1.570\,796\,326\,79$
A_{floor}	$= s^2 = \frac{\pi^2 h^2}{4}$	$\approx h^2 \cdot 2.467\,401\,100\,27$
$A_{face} = \frac{s^2}{4}\sqrt{1+\frac{4h^2}{s^2}} = \frac{s^2}{4}\cdot\sqrt{1+\frac{4h^2}{\frac{\pi^2h^2}{4}}}$	$= \frac{s^2}{4}\cdot\sqrt{1+\frac{16}{\pi^2}}$	$\approx s^2 \cdot 0.404\,748\,296\,65$
	$= \frac{\pi^2h^2}{16}\cdot\sqrt{1+\frac{16}{\pi^2}}$	$\approx h^2 \cdot 0.998\,676\,392\,49$
$y = \frac{s}{2}\cdot\sqrt{1+\frac{4h^2}{s^2}}$	$= \frac{s}{2}\cdot\sqrt{1+\frac{16}{\pi^2}}$	$\approx s \cdot 0.809\,496\,593\,3$
	$= \frac{\pi h}{4}\cdot\sqrt{1+\frac{16}{\pi^2}}$	$\approx h \cdot 1.271\,554\,275\,31$
$r = \sqrt{h^2+\frac{1}{2}s^2} = \sqrt{\frac{4s^2}{\pi^2}+\frac{1}{2}s^2}$	$= s\cdot\sqrt{\frac{1}{2}+\frac{4}{\pi^2}}$	$\approx s \cdot 0.951\,464\,520\,92$
	$= \frac{\pi h}{2}\cdot\sqrt{\frac{1}{2}+\frac{4}{\pi^2}}$	$\approx h \cdot 1.494\,556\,974\,54$
$\alpha = \arctan\frac{2h}{s}$	$= \arctan\frac{4}{\pi}$	$\approx 51.853\,974°$
$\beta = \arctan\sqrt{1+\frac{4h^2}{s^2}}$	$= \arctan\sqrt{1+\frac{16}{\pi^2}}$	$\approx 58.297\,709\,1°$
$\gamma = \arctan\frac{h\sqrt{2}}{s}$	$= \arctan\frac{2\sqrt{2}}{\pi}$	$\approx 41.997\,223\,9°$

<u>"Circlamid":</u> (i.e. $\Gamma = 6$, I invented this name because a circle fits exactly 6 times around itself)

Γ	$= 6$	$= 6$
h	$= \frac{2}{3}s$	$\approx s \cdot 0.666\,666\,666\,67$
s	$= \frac{3}{2}h$	$= h \cdot 1.5$
A_{floor}	$= s^2 = \frac{9}{4}h^2$	$= h^2 \cdot 2.25$
$A_{face} = \frac{s^2}{4}\sqrt{1+\frac{4h^2}{s^2}} = \frac{s^2}{4}\cdot\sqrt{1+\frac{4\cdot 2^2}{3^2}} = \frac{s^2}{4}\cdot\sqrt{\frac{25}{9}}$	$= \frac{5s^2}{12}$	$\approx s^2 \cdot 0.416\,666\,666\,67$
$= \frac{9h^2}{4\cdot 4}\cdot\frac{5}{3}$	$= \frac{15h^2}{16}$	$= h^2 \cdot 0.937\,5$
$y = \frac{s}{2}\cdot\sqrt{1+\frac{4h^2}{s^2}} = \frac{s}{2}\cdot\sqrt{1+\frac{16}{3^2}} = \frac{s}{2}\cdot\sqrt{\frac{25}{9}}$	$= \frac{5}{6}s$	$\approx s \cdot 0.833\,333\,333\,33$
	$= \frac{5}{4}h$	$= h \cdot 1.25$
$r = \sqrt{h^2+\frac{1}{2}s^2} = \sqrt{\frac{4s^2}{3^2}+\frac{1}{2}s^2} = s\cdot\sqrt{\frac{4}{9}+\frac{1}{2}}$	$= s\cdot\sqrt{\frac{17}{18}}$	$\approx s \cdot 0.971\,825\,315\,8$
	$= \frac{3}{2}h\cdot\sqrt{\frac{17}{18}}$	$\approx h \cdot 1.457\,737\,973\,71$
$\alpha = \arctan\frac{2h}{s}$	$= \arctan\frac{4}{3}$	$\approx 53.130\,102\,4°$
$\beta = \arctan\sqrt{1+\frac{4h^2}{s^2}} = \arctan\sqrt{1+\frac{16}{9}}$	$= \arctan\frac{5}{3}$	$\approx 59.036\,243\,5°$
$\gamma = \arctan\frac{h\sqrt{2}}{s}$	$= \arctan\frac{2\sqrt{2}}{3}$	$\approx 43.313\,856\,7°$

Note: $\frac{s}{2} : h : y = 3 : 4 : 5$ which is the first Pythagorean triple

$\therefore\ h : y : s = 4 : 5 : 6$

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Equilateral pyramid:

	r	$= s$	
	$\therefore y = \sqrt{r^2 - \left(\frac{s}{2}\right)^2} = \sqrt{s^2 - \frac{s^2}{4}} = s \cdot \sqrt{3/4}$	$= \frac{s}{2}\sqrt{3}$	
so:	$h = \sqrt{y^2 - \left(\frac{s}{2}\right)^2} = \sqrt{s^2 - \frac{s^2}{4} - \frac{s^2}{4}} = s \cdot \sqrt{1/2}$	$= s/\sqrt{2}$	$\approx s \cdot 0.707\,106\,781\,18$
	s	$= h \cdot \sqrt{2}$	$\approx h \cdot 1.414\,213\,562\,37$
	$\Gamma = \frac{4s}{h}$	$= 4 \cdot \sqrt{2}$	$\approx 5.656\,854\,249\,49$
	A_{floor}	$= s^2 = 2h^2$	
	$A_{face} = \frac{sy}{2}$	$= \frac{s^2\sqrt{3}}{4}$	$\approx s^2 \cdot 0.433\,012\,701\,89$
		$= \frac{h^2\sqrt{3}}{2}$	$\approx h^2 \cdot 0.866\,025\,403\,78$
	$y =$	$= \frac{s}{2}\sqrt{3}$	$\approx s \cdot 0.866\,025\,403\,78$
		$= h \cdot \sqrt{3/2}$	$\approx h \cdot 1.224\,744\,871\,39$
	r	$= s = h \cdot \sqrt{2}$	$\approx h \cdot 1.414\,213\,562\,37$
	$\alpha = \arctan\frac{2h}{s}$	$= \arctan\sqrt{2}$	$\approx 54.735\,610\,3°$
	$\beta = \arctan\sqrt{1 + \frac{4h^2}{s^2}}$	$= \arctan\sqrt{3}$	$= 60°$
	$\gamma = \arctan\frac{h\sqrt{2}}{s}$	$= \arctan 1$	$= 45°$

It should be obvious that all corresponding figures nearly equal one another for each type of pyramid, and the *base area* of a quamispheramid is just a tiny bit smaller than that of a golden pyramid. With the calculated values of β, the faces approximate equilateral triangles, but they are definitely not (except of course the equilateral pyramid). Apart from the actual values of *side* and *height*, the p/h ratio fully defines a right sqaure pyramid.

Now consider the Cheops/Khufu pyramid. It does have indented *sides* and its *circumference* does approximately equal 2π times its *height*. Might it be a golden quamispheramid?

On https://en.wikipedia.org/wiki/Great_Pyramid_of_Giza we find

its original *height* as:	$h = 280$ Egyptian Royal cubits $= 146.7$ metres
and its *base* (from corner to corner) is:	$s = 440$ Egyptian Royal cubits $= 230.34$ metres
(an Egyptian Royal cubit is:	52.4 ± 0.1 centimetres)

and on https://nl.wikipedia.org/wiki/Piramide_van_Cheops we find that

its *indentation* approximates:	60 centimetres
or:	$\frac{0.6}{230} \approx 2.6‰$

In http://www.thegreatpyramidofgiza.ca/book/TheGreatPyramidofGIZA.pdf (page 18) we find

inner square circuit:	921.455 metres
outer square circuit:	928.7369 metres
leading to an *indentation* of:	$\frac{928.7369-921.455}{8} \approx 91$ centimetres
yielding a relative *indentation* of:	$\frac{(928.7369-921.455)/8}{928.7369/4} \approx 3.9‰$

The Cheops pyramid should be indented by just 11 centimetres to be a true golden quamispheramid, but its actual indentation is roughly 8 times more.

However, if we calculate the ratio of half the *circumference* to the *height*,

we obtain: $\frac{2s}{h} = \frac{2 \cdot 440}{280} = \frac{22}{7}$ (using the measures in cubits)

which happens to be the upper limit of π as found by Archimedes (287 - 212 BCE).

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If we also approximate: $\varphi \approx \frac{8}{5}$

we obtain: $i = 230.34 \cdot \left(\frac{1}{2} - \frac{22/7}{8}\sqrt{\frac{8}{5}}\right) \approx 0.7073$ metres ≈ 71 centimetres.

Cf. $\frac{1}{2}\sqrt{2} \approx 0.7071$, which merely is a mathematical coincidence, but please note the original definition of the metre was one ten millionth of the *distance* from the North Pole to the equator, so these centimetres indeed have something to do with Earth's *circumference* which nearly equals $12 \times 60 \times 60$ times the pyramid's *circumference*, so one might say the square root of two is contained in the pyramid in another way than just by its diagonal. But this is just a mathematical triviality that follows from the $\pi = \frac{22}{7}$ and $\varphi = \frac{8}{5}$ approximations in combination with a base of 230.34 metres which could be related to Earth's *circumference*.

I dare to state the pyramid builders did have knowledge of both $\pi \approx \frac{22}{7}$ and $\varphi \approx \frac{8}{5}$. They must also have known that the geometry based on π fits just inside the one based on φ.

Some other facts about the Cheops pyramid:

Alignment: north-east-south-west with *deviation* $\leq 0.01°$

The southward shafts point to the culminations of Sirus and Alnitak (the eastmost star of Orion's belt, or the leftmost as seen from the north) as they were around 2450 BCE.

Circumference: $\approx$ Earth's *circumference*$/(12 \times 60 \times 60)$

Location: $29°58'45''.0$ N, $31°08'00''.8$ E

so its *latitude* nearly equals: $30°$ and then the celestial equator

has a southmost *altitude* of: $60°$ which is the *angle* that divides a circle into 6 equal parts. A circle fits exactly 6 times around itself, so $60°$ is some kind of a special *angle* that could be used to indicate something is round, be it circular or spherical.

Yet another fact: according to https://en.wikipedia.org/wiki/Pyramid_of_Menkaure the third pyramid at Giza, which is that of Menkaure, measures a corner-to-corner distance of 200 cubits and a height of 125. Their ratio equals $\frac{8}{5}$, which happens to be the aforementioned approximation of φ, which - as explained - matches Cheops's corners. The southeast corner of Cheops lies quite accurately at the extended diagonal of Menkaure. Menkaure's northwest corner lies at the line that connects the apices of Cheops and Chephren.

Many people see mystical things in many aspects of the Cheops pyramid, for example the *surface area* of a face being equal to the square of its *height*. As derived above, this is an implicit aspect of a golden pyramid and a quamispheramid automatically approaches it, so it is a free gift. Nothing special. A mathematical triviality. Also the fact that both π and φ appear in its geometry is nothing special. If you take one, you implicitly get the other because $\frac{\pi}{4}\sqrt{\varphi} \approx 0.999\,042$. This is a mathematical coincidence, which does certainly not mean it is by chance and it definitely is not due to the cleverness of the pyramid builders that both π and φ appear in the pyramid with such a close match. It is just and simply a mathematical fact that $\frac{\pi}{4}\sqrt{\varphi}$ nearly equals 1, just like $2 + 3 = 5$, or 1% being exactly one hundredth of the whole thing. Nothing mysterious.

What is interesting:

- The pyramid more or less marks the southmost point of the Nile Delta whilst Alexandria approximates the western edge of the Nile Delta and Port Said is near its eastern edge;
- the *distance* from the pyramid to the nearest point of the coast at Alexandria, 7 km to the southwest of the location of the Pharos, the old Lighthouse of Alexandria, measures 179.7 km;
- this line quite accurately matches the northwest - southeast direction, so it extends a diagonal of the pyramid;

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- the other extended diagonal meets the current mediterranean coast slightly East of the Port Said East Power Plant, where the coast is nearest to the pyramid at a *distance* of 182.1 km;
- the distance between these coastal points equals 255.8 km;
- the enclosed *angle* of these extended diagonals equals: $\arccos\frac{179.7^2+182.1^2-255.8^2}{2\cdot179.7\cdot182.1} \approx 89.98°$;
- the line from the pyramid practically straight to the north meets the current mediterranean coast at a distance of 180.6 km at its (locally) farthest point from the pyramid, so the pyramid is very near the centre of the (circumscribed) circle defined by these 3 coastal points. The pyramid's diagonals mark a quarter of a circle that encloses practically the entire Nile Delta.

By the way, did you know a branching river mouth is named a *delta* because as seen from north to south the fertile area around the final branches of the Nile looks like the Greek capital "Δ"?

For the second greatest pyramid at Giza, the pyramid of Chephren/Khafre, https://en.wikipedia.org/wiki/Pyramid_of_Khafre says:

location:	$29°58'33.''5$ N, $31°07'48.''4$ E
original *height*:	274 cubits $= 143.5$ m
height difference with Cheops:	6 cubits
base:	411 cubits $= 215.25$ m
circumference over *height* ratio:	$\frac{4\cdot411}{274} =$ exactly 6.

As already mentioned, the number 6 "belongs" to a circle, which can be perfectly surrounded by exactly 6 identical circles, so Chephren is what I called a "circlamid" whilst Cheops is a "golden quamispheramid" which reflects both a hemisphere (and thus a sphere) and the Golden Ratio.

Let's take a closer look at such a pyramid.

Assume a normalised *height* of:	1
then its *circumference* equals:	6
hence its *base* is:	$\frac{6}{4} = \frac{3}{2}$
so the midpoint of the *base* is at:	$\frac{3}{4}$
yielding a mid-base to top *distance* of:	$\sqrt{\left(\frac{3}{4}\right)^2 + 1^2} = \sqrt{\frac{9}{16}+\frac{16}{16}} = \sqrt{\frac{25}{16}} = \frac{5}{4}$
and then each ridge equals:	$\sqrt{\left(\frac{3}{4}\right)^2 + \left(\frac{5}{4}\right)^2} = \sqrt{\frac{34}{16}} = \sqrt{\frac{17}{8}} \approx 1.45774 < \frac{3}{2}$
The *base angles* are:	$\arctan\frac{5/4}{3/4} \approx 59.0362435°$ each
and then the *top angle* equals:	$180 - 2 \times$ that $= 61.927513°$

But please do not conclude something like: *Wow, they made a nearly perfect equilateral triangle as well!* They didn't do it "as well", they got it as a free gift. It simply is a mathematical fact that this *base angle* equals $59.04°$ if the *circumference* equals 6 times the *height*, and it is not an equilateral triangle. It's close, but no cigar.

The ratio of the sides of the triangle from mid-base via the centre of the pyramid's floor to its top equals: $\frac{3}{4}:1:\frac{5}{4} = \frac{3}{4}:\frac{4}{4}:\frac{5}{4} = 3:4:5$

which is the smallest Pythagorean triple, but this too is a free gift from mathematics, not a clever achievement of the pyramid builders.

The *slope* of the ridges from the corners to the top equals: $\arctan\frac{1}{\frac{3}{4}\sqrt{2}} = \arctan\left(\sqrt{8/9}\right) \approx 43.314°$

Note: as seen from Cheops, Chephren is at a west-of-south bearing of: $\sim 43.275°$

The diagonals of Cheops and Chephren are not on the same line.

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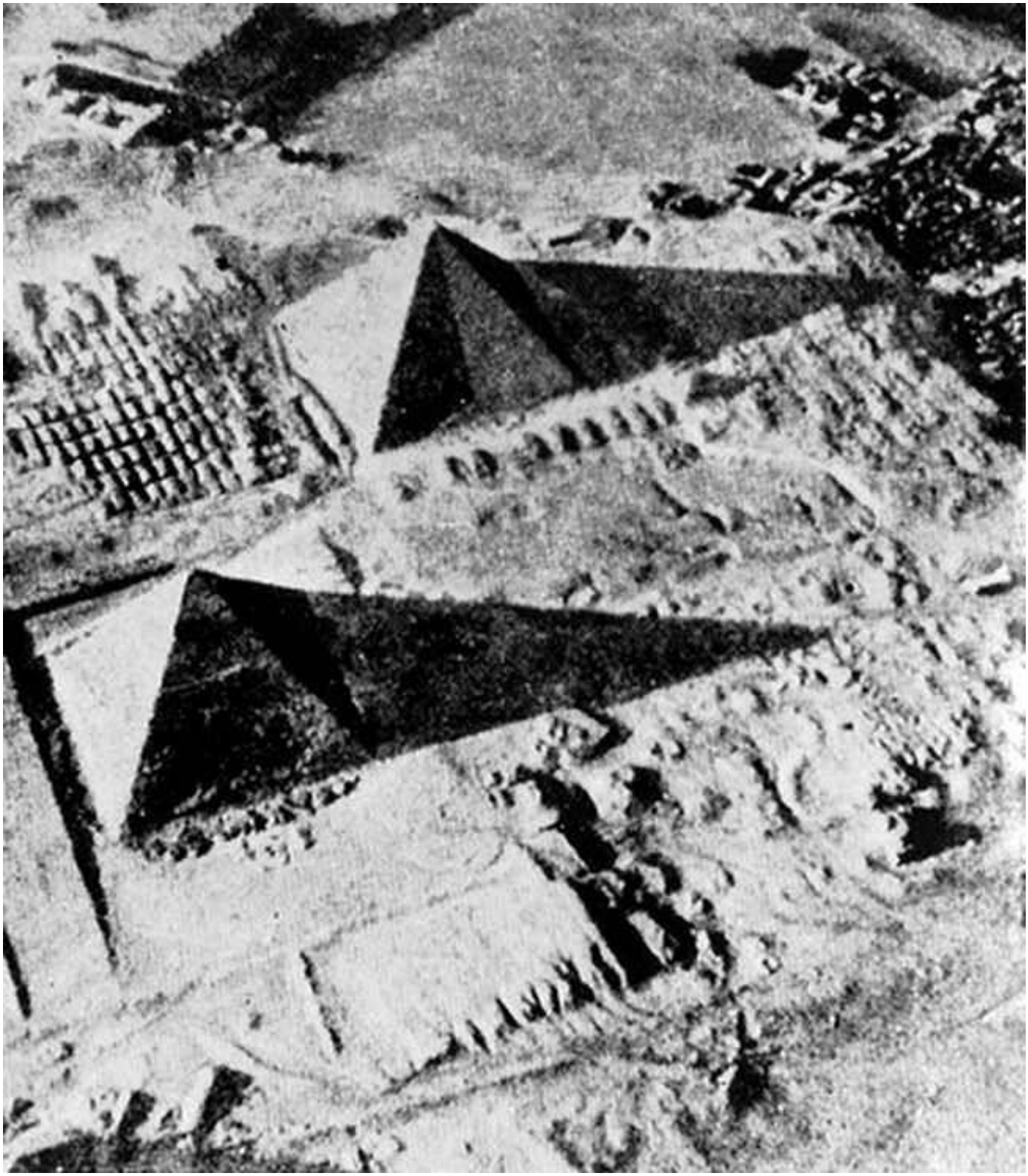
It seems Chephren (lower left) does not have indented faces like Cheops (top middle) has:



https://www.stolenhistory.org/attachments/huit-face-pyramide-kheops-egypte-03-jpg.7744/
https://www.stolenhistory.org/threads/single-photo-8-sided-egyptian-pyramid.351/

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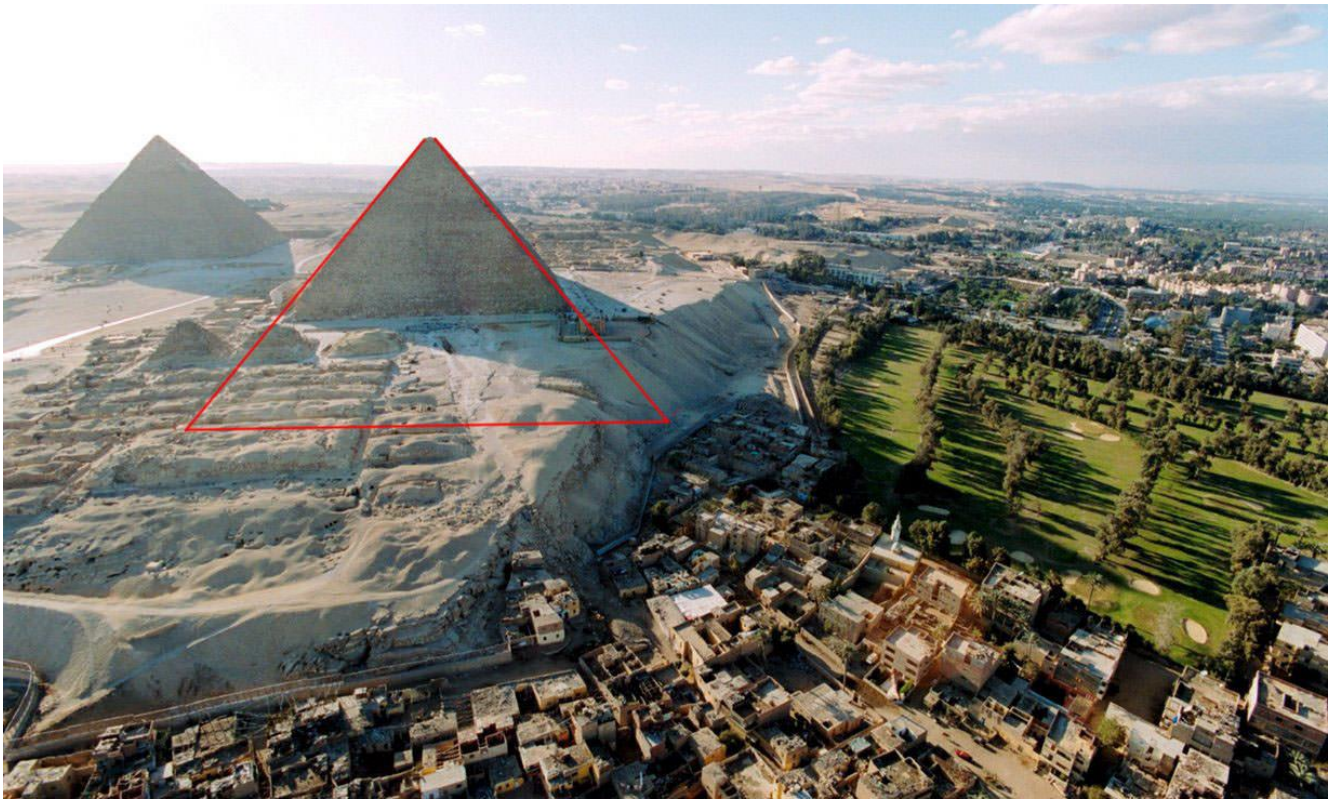
Doesn't the Giza plateau look a bit like a flood deposit?

https://66.media.tumblr.com/tumblr_lpn57rjvBi1qi6b7vo1_500.png
https://ro.pinterest.com/pin/420664421426383553/?nic_v1=1a5hAx2aKnd6Ap8ziO8C83TFX2Qn%2BR0Co%2FKj52OY%2BPbn2Enrvez53yMZO9DsVFhQId

To my opinion the red triangle as an extension of the Cheops pyramid below is a false assumption.



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Height of the plateau with respect to the Desert Park (the green area): ca. 40 metres.

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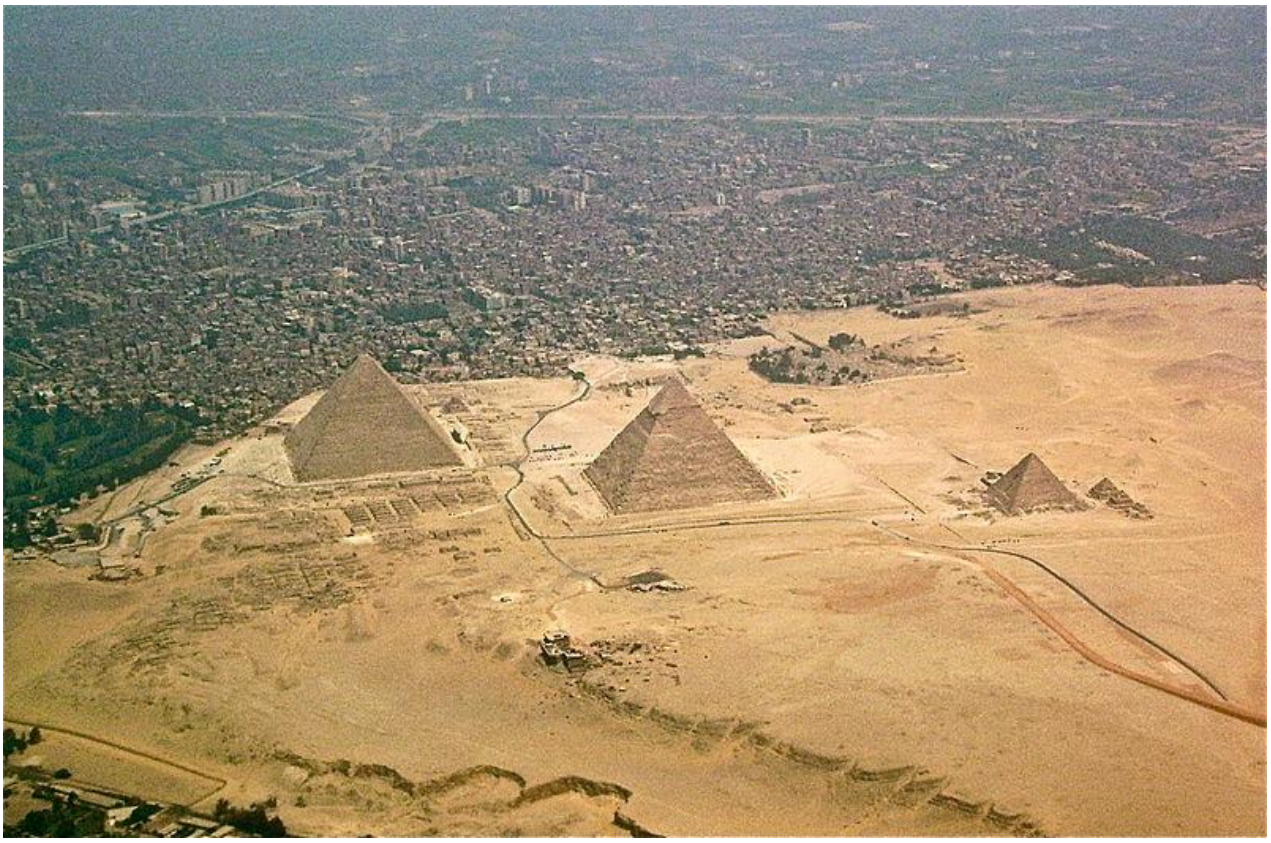

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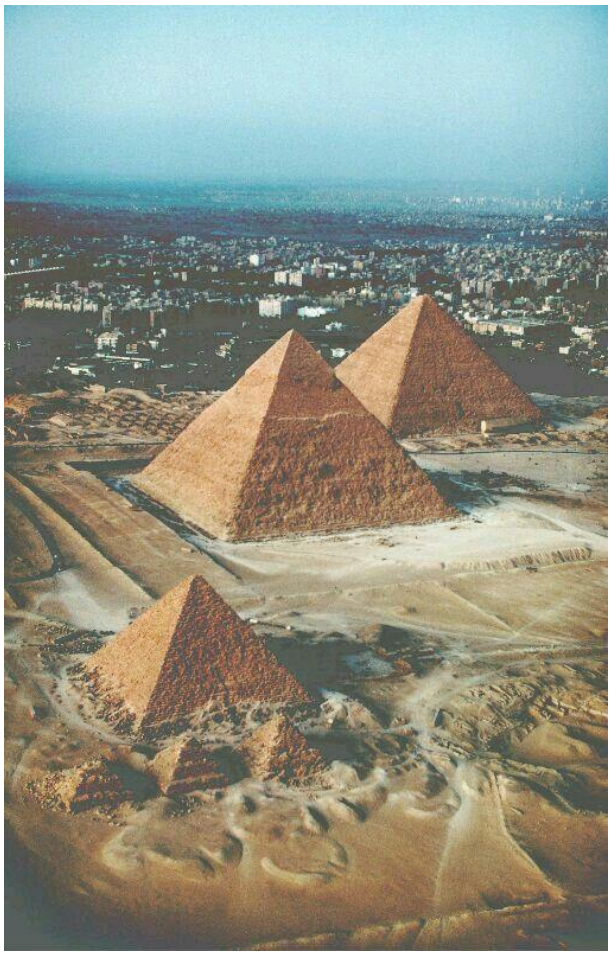

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