Near the end of Einstein's very first publication of his ~~theory~~ conclusion of relativity, we find:

$$E_{\text{kin}} = mc^2\left(\frac{1}{\sqrt{1-v^2/c^2}} - 1\right)$$

Kinetic energy (*energy* of motion) as measured by a stationary observer equals a body's *mass* (measured when it is not moving) times the *speed of light* squared times the so-called *Lorentz factor* minus one. If you're not familiar with the latter, then please take this for granted, its explanation is irrelevant in the current context. "v" is the body's *velocity* and "c" is the *speed of light*. With the above, we can do some simple mathematics and since the *mass* was measured at rest, we'll give it a corresponding suffix and call it the ***rest mass***:

$$E_{\text{kin}} = \frac{m_{\text{rest}}c^2}{\sqrt{1-v^2/c^2}} - m_{\text{rest}}c^2$$

is the same as:

$$m_{\text{rest}}c^2 + E_{\text{kin}} = \frac{m_{\text{rest}}c^2}{\sqrt{1-v^2/c^2}}$$

The actual meaning of $m_{\text{rest}}c^2$ seems still unclear, but it should be obvious that it must be ***some*** type of *energy*. We will call it the ***rest energy***:

$$\boldsymbol{E_{\text{rest}} = m_{\text{rest}}c^2}$$

which we incorporate in the ***total energy***:

$$E_{\text{tot}} = E_{\text{rest}} + E_{\text{kin}} = \frac{m_{\text{rest}}c^2}{\sqrt{1-v^2/c^2}}$$

Squaring this yields:

$$E_{\text{tot}}^2 = \frac{m_{\text{rest}}^2c^4}{1-v^2/c^2} = \frac{E_{\text{rest}}^2}{1-v^2/c^2}$$

which we

$$E_{\text{tot}}^2\left(1 - \frac{v^2}{c^2}\right) = E_{\text{rest}}^2$$

rewrite

$$E_{\text{tot}}^2 - E_{\text{tot}}^2\frac{v^2}{c^2} = E_{\text{rest}}^2$$

step by

$$E_{\text{tot}}^2 = E_{\text{rest}}^2 + E_{\text{tot}}^2\frac{v^2}{c^2}$$

step to:

$$E_{\text{tot}}^2 = E_{\text{rest}}^2 + \frac{E_{\text{tot}}^2}{c^4}v^2c^2$$

Similar to $E_{\text{rest}} = m_{\text{rest}}c^2$, we can state: $\boldsymbol{E_{\text{tot}} = m_{\text{tot}}c^2}$ and we will call $\boldsymbol{m_{\text{tot}}}$ the ***total mass***.

therefore:

$$\boldsymbol{E_{\text{tot}}^2 = m_{\text{rest}}^2c^4 + m_{\text{tot}}^2v^2c^2}$$

Together with:

$$E_{\text{tot}}^2 = m_{\text{tot}}^2c^4$$

we obtain:

$$m_{\text{tot}}^2c^4 = m_{\text{rest}}^2c^4 + m_{\text{tot}}^2v^2c^2$$

or:

$$m_{\text{tot}}^2c^4 - m_{\text{tot}}^2v^2c^2 = m_{\text{rest}}^2c^4$$

Division by c^4:

$$m_{\text{tot}}^2 - m_{\text{tot}}^2v^2/c^2 = m_{\text{rest}}^2$$

renders:

$$m_{\text{tot}}^2(1 - v^2/c^2) = m_{\text{rest}}^2$$

hence:

$$\boldsymbol{m_{\text{tot}}^2 = m_{\text{rest}}^2/(1 - v^2/c^2)}$$

Similar to:

$$E_{\text{tot}} = E_{\text{rest}} + E_{\text{kin}}$$

we can write:

$$m_{\text{tot}} = m_{\text{rest}} + m_{\text{kin}}$$

We already have:

$$E_{\text{rest}} = m_{\text{rest}}c^2$$

as well as:

$$E_{\text{tot}} = m_{\text{tot}}c^2$$

so it must be that:

$$E_{\text{kin}} = m_{\text{kin}}c^2$$

altogether, we find:

$$\boldsymbol{m_{\text{tot}} = m_{\text{rest}} + m_{\text{kin}} = m_{\text{rest}}/\sqrt{1 - v^2/c^2}}$$

Multiplied by c^2:

$$\boldsymbol{E_{\text{tot}} = E_{\text{rest}} + E_{\text{kin}} = E_{\text{rest}}/\sqrt{1 - v^2/c^2}}$$

Adding *energy* behaves like adding *mass*! In this context, *mass* should be seen as *inertia*, the resistence against change of motion. *Energy* appears to have *inertia* too! However, this is only observed by a stationary observer. The moving body does not experience this in its own frame of reference.

In the found: $E_{\text{tot}}^2 = m_{\text{tot}} c^2 = m_{\text{rest}}^2 c^4 + m_{\text{tot}}^2 v^2 c^2$

we can substitute: $m_{\text{tot}} v = m_{\text{rest}} v / \sqrt{1 - v^2/c^2} = p_{\text{class}} / \sqrt{1 - v^2/c^2} = p_{\text{tot}}$

where p_{class} is the *momentum* as defined in classical mechanics. Then p_{tot} is of course the ***total momentum*** of the moving body, which in practice is called the ***relativistic momentum*** (the *total mass* is actually called *relativistic* as well).

Therefore: $\boldsymbol{E_{\text{tot}}^2 = m_{\text{tot}} c^2 = m_{\text{rest}}^2 c^4 + p_{\text{tot}}^2 c^2}$

If we assign a *mass* to a photon: $E_\gamma = m_\gamma c^2 \therefore m_\gamma = E_\gamma / c^2$

then its *rest mass*: $m_{\text{rest},\gamma} = m_\gamma \sqrt{1 - \frac{(v_\gamma = c)^2}{c^2}} = 0$ (although a photon can't be at rest)

equals nought, hence: $E_{\text{tot}}^2 = p_{\text{tot}}^2 c^2$

After replacing the suffix, we find: $E_\gamma = p_\gamma c$

Einstein got his Nobel Prize for: $E_\gamma = h\nu$ (please note: "ν" is the greek letter *nu*)

so **a photon's *momentum*** equals: $\boldsymbol{p_\gamma = h\nu / c}$

Above, we found: $E_{\text{rest/tot/kin}} = m_{\text{rest/tot/kin}} c^2$

which in general becomes: $\boldsymbol{E = mc^2}$

This is often called the most famous equation of all times. In his very next publication, Einstein demonstrates that a radiating body must lose *mass* according to this equation. It suddenly explained how the sun is able to shine so luminously for billions of years! In its interior, the sun actually converts *mass* to *energy*. It is also what drives nuclear power plants and unfortunately, we must add atomic bombs to this list too. Annihilation reactions of elementary particles and their antiparticles show that the entire *mass* of a particle can be converted to *energy* according to Einstein's formula. *Mass* is in fact a condensed form of *energy*.

The speed of light is: $c = 299\,792\,458$ m/s ≈ 1 billion (Dutch: 1 miljard) km/h. When we square that, we obtain $89\,875\,517\,873\,681\,764 \approx 10^{17}$ m²/s². This means that even a very small *mass* could be converted to a very **HUGE** amount of *energy*!

Example 1: The sun continuously emits $3.828 \times 10^{26} = 382\,800\,000\,000\,000\,000\,000\,000\,000$ watts of power in all directions. That corresponds to $1.344 \times 10^{17} = 134\,400\,000\,000\,000\,000$ kilograms per year or 1 earth mass in 44.4 million years, 225 earth masses in 10 billion (Dutch: 10 miljard) years. The total mass of the sun is 333 000 times that of the earth, so the sun will "burn" merely $0.00068 \approx \frac{2}{3}$ ‰ of its total mass during its entire lifetime.

Example 2: In (actually above) Hiroshima, 0.75 g was converted to *energy* and in Nagasaki it was 1 g. This 1 gram yielded 90 million megajoules of energy. An electric car (as used on average in The Netherlands) consumes roughly 2.5 MJ/km, so this single gram would suffice for 36 million kilometres, which is roughly 100 times the distance to the moon or one quarter of the distance to the sun.

A Mr. Taylor has found that: $f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x - a)^n$

If x is small we can say: $\boldsymbol{\frac{1}{\sqrt{1-x^2}} = 1 + \frac{x^2}{2}} + \frac{3x^4}{8} + \frac{3x^6}{16} + \mathcal{O}(x^8)$

hence: $\boldsymbol{E_{\text{kin}}} = mc^2 \left(\frac{1}{\sqrt{1 - v^2/c^2}} - 1 \right) \approx mc^2 \left(1 + \frac{v^2/c^2}{2} - 1 \right) = \boldsymbol{\frac{1}{2} m v^2}$

which is the classical equation for *kinetic energy*. It applies if $v \ll c$, i.e. in daily life.