Classical thermodynamics yields: $E_{class} = \frac{3}{2}kT_{class} = \frac{1}{2}mv^2 = mc^2\frac{\beta^2}{2}$

Relativistic *kinetic energy*: $E_{rel} = \gamma m_0c^2 - m_0c^2 = m_0c^2(\gamma - 1) = m_0c^2\left(\frac{\beta^2}{2} + \sigma(\beta^4)\right)$

hence: $\gamma = 1 + \frac{E_{rel}}{m_0c^2}$

yielding what I'll call a *relativistic temperature* of:

$$T_{rel} = \frac{2E}{3k} = \frac{2m_0c^2(\gamma-1)}{3k}$$

where γ of course relates to the *thermal velocity* of the molecules.

This so found *relativistic temperature* would be able to reach infinity, but I think physical limitlessness is fundamentally impossible, so I ~~define~~ pick from thin air what I will call the

relativistic Boltzmann constant: $\boldsymbol{k_{rel} \coloneqq \gamma k}$

just in order to investigate what it could imply.

It renders: $T_{rel} = \frac{2m_0c^2(\gamma-1)}{3k_{rel}} = \frac{2m_0c^2}{3k} \cdot \frac{(\gamma-1)}{\gamma}$

yielding: $T_{max} = \lim_{\gamma\to\infty} T_{rel} = \frac{2m_0c^2}{3k}$

This would be the *temperature* of an ideal gas where each molecule would have a *thermal energy* equal to its own relativistic *mass equivalent* and it would be impossible to become hotter.

For $m = 1$ Da we obtain: $T_{max,1u} = \frac{2}{3k} \cdot (1\ \text{Da}) \cdot c^2 \approx 7.206 \times 10^{12}$ K.

This value of $m = 1$ Da roughly applies to monatomic hydrogen, of which the universe mainly consists. The lightest hadron is π^0 with a *mass* of $134.8766\ \text{MeV}/c^2 \approx 0.14479598$ Da, yielding: $T_{max,\pi^0} = \frac{2}{3k} \cdot 134.8766\ \text{MeV} \approx 1.043 \times 10^{12}$ K. For a hypothetical gas consisting of neutrons (ignoring their decay) we find: $T_{max,n} \approx 7.269 \times 10^{12}$ K. I think that more massive molecules cannot exist for a longer *time* at such *temperatures*. They will be blown apart by the collisions or decay very rapidly. Practically all known particles are unstable with *lifetimes* of: neutron: 15 min which I'll call quasi stable, muon: 2.2 µs (in which light travels just 660 m), all others: 52 ns or (far) less, which means they can hardly be called "existing".

These T_{max} values are all similiar to the *Hagedorn temperature*[1] (a sort of "melting point" where spontaneous pair production occurs): $T_H = 158$ MeV or 1.222×10^{12} K. WikipediA[2] gives 150 MeV and 1.7×10^{12} K (which cannot be correct since $150\ \text{MeV} \times \frac{2}{3k} = 1.16 \times 10^{12}$ K).

Can it still be called a rise in *temperature* if addition of *energy* no longer results in more movement of the particles but in more moving particles? The term *temperature* applies only to the stochastical movement of molecules relative to one another or to their common barycentre. I do not consider it a *temperature* if molecules or whatever particles are travelling together at a high speed, like for example the 7 TeV that the LHC pumps into each proton which would correspond to 5.4×10^{16} K.

We've got: $T_{rel} = \frac{2m_0c^2}{3k} \cdot \frac{(\gamma-1)}{\gamma} = \frac{2m_0c^2}{3k} \cdot \left(1 - \sqrt{1-\beta^2}\right)$

for small values of β: $T_{rel} \approx \frac{2m_0c^2}{3k} \cdot \frac{\beta^2}{2} = \frac{2}{3k} \cdot \frac{1}{2}m_0v^2 = T_{class}$

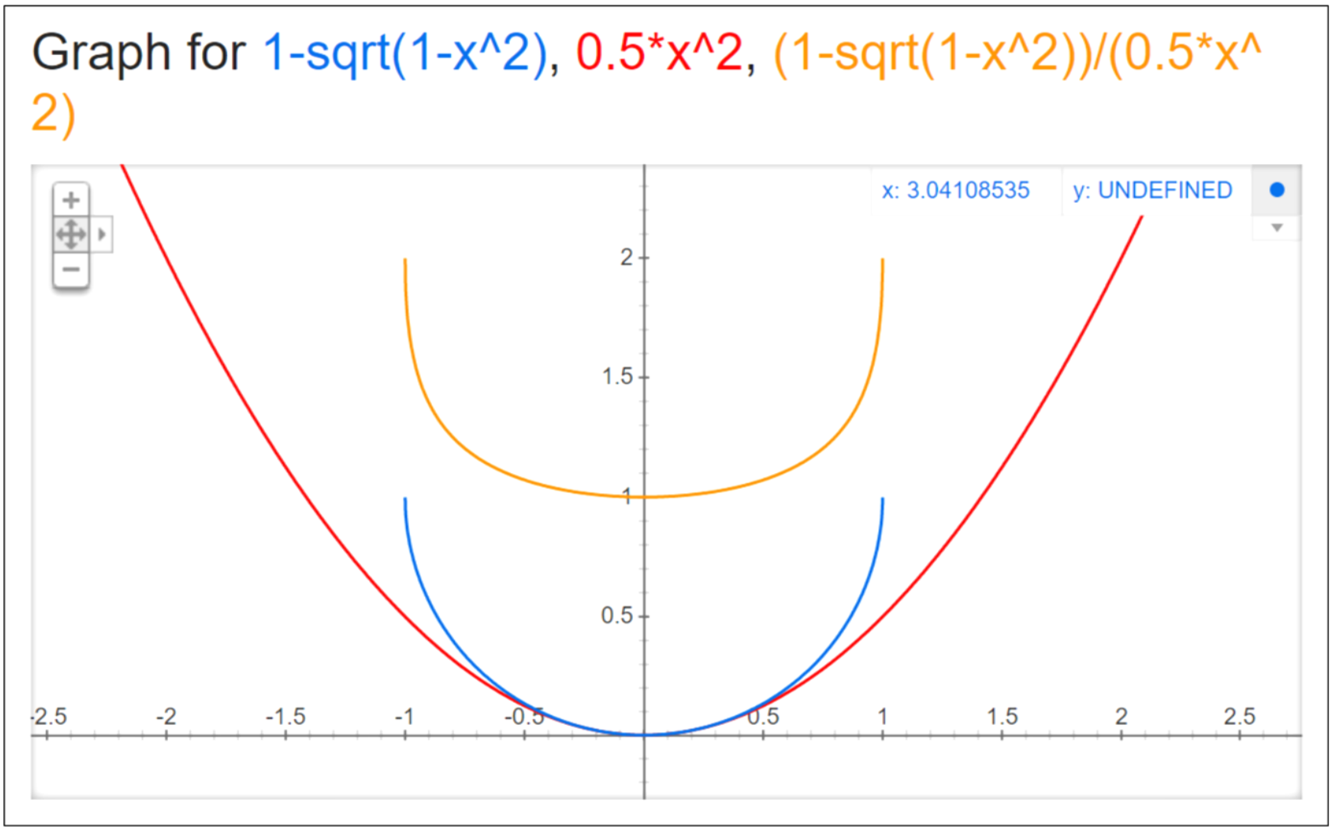
[1] https://cerncourier.com/a/the-tale-of-the-hagedorn-temperature/
[2] https://en.wikipedia.org/wiki/Hagedorn_temperature (as of 2021-01-10)

hence: $$\beta^2 = \frac{3k}{m_0 c^2} T_{class}$$

and: $$\frac{T_{rel}}{T_{class}} = \frac{1-\sqrt{1-\beta^2}}{\beta^2/2} \approx 1 + \frac{\beta^2}{4} + \mathcal{O}(\beta^4) \;\therefore\; \Delta T = \frac{\beta^2}{4} T_{class} = \frac{3k}{4 m_0 c^2} T_{class}^2$$

For molecular hydrogen H_2 at $T = 3\,000$ K (roughly the cosmological recombination temperature, so I'm not sure if H_2 can abundantly exist at that temperature, but the H_2 *bond energy* of 4.52 eV corresponds to $T = 4.52\text{ eV} \cdot \frac{2}{3k} \approx 35\,000$ K) we have $m_0 = 2.016$ Da yielding $\Delta T = 0.31$ μK, which would be the accuracy required to measure any effect at all and I **think** we are unable to achieve that at $T = 3\,000$ K.

Altogether it seems that the (to my opinion rather elegant) idea of a *relativistic Boltmann constant* does not contradict current theories. It is in agreement with the *Hagedorn temperature* and it avoids physical unlimitedness. But I picked it from thin air.



Dimensionless T_{rel}, T_{class}, and their ratio as function of the *thermal velocity* β (graph by Google).

T_{rel} is half a circle (just like the *Lorentz contraction* if drawn with an aspect ratio of 1:1), whilst T_{class} is parabolic.

Ludwig Boltzmann (1844-1906)