Gravitational potential energy: $V = V_\infty - \frac{GMm}{r}$ [1]

choose: $V_\infty = 0$ [2]

define *gravitational potential*: $U = \frac{V}{m} = \frac{-GM}{r}$ [3]

Escape velocity: $v_e = \sqrt{\frac{2GM}{r}}$ $\quad \therefore v_e^2 = -2U \quad \therefore \frac{2U}{c^2} = \frac{-v_e^2}{c^2} = \frac{-2GM}{rc^2}$ [4]

Kinetic energy: $T = \frac{1}{2}mv^2$ [5]

presume: $T_\infty = 0$ [6]

define "*kinetic potential*": $S = \frac{T}{m} = \frac{1}{2}v^2$ $\quad \therefore v^2 = 2S \quad \therefore \frac{2S}{c^2} = \frac{v^2}{c^2}$ [7]

Conservation of *energy*: $T + V = T_\infty + V_\infty = 0$ [8]

hence: $T = -V$ $\quad \therefore S = -U$ [9]

therefore: $\frac{2S}{c^2} = \frac{-2U}{c^2}$ [10]

and then: $\sqrt{1 - \frac{2S}{c^2}} = \sqrt{1 + \frac{2U}{c^2}}$ [11]

so, using [4] and [7]: $\sqrt{1 - \frac{v^2}{c^2}} = \sqrt{1 - \frac{v_e^2}{c^2}} = \sqrt{1 - \frac{2GM}{rc^2}}$ [12]

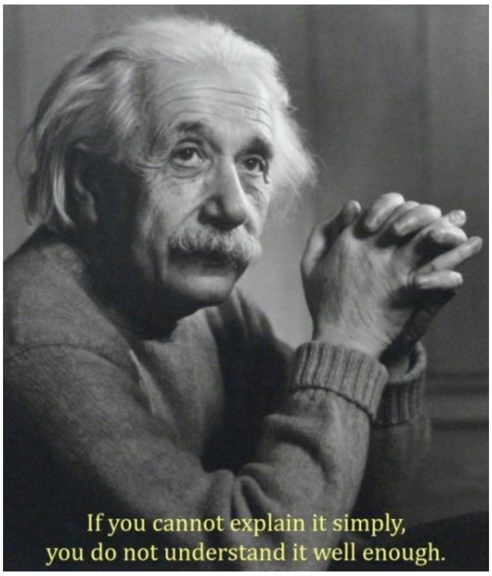
We have from SR: $\sqrt{1 - \frac{v^2}{c^2}} = \frac{1}{\gamma}$ = the reciprocal *Lorentz factor* [13]

and from GR: $\sqrt{1 - \frac{v_e^2}{c^2}} = \sqrt{1 - \frac{2GM}{rc^2}}$ = the "*Schwarzschild factor*" [14]

It appears that the reciprocal *Lorentz factor* of a freely falling body equals the *gravitational length contraction* factor as follows from the Schwarzschild solution. This equality simply follows from the conservation of *energy*. Without General Relativity. No tensor calculus. We found the *Schwarzschild factor* using classical mechanics and Special Relativity only.

Since the reciprocal *Lorentz factor* depends on the *velocity* which results from *acceleration* and the *Schwarzschild factor* results from *gravitation*, we arrived at Einstein's *equivalence principle* that says *acceleration* and *gravitation* are indistinguishable.

Apparently, conservation of *energy* and Einstein's *equivalence principle* are fundamentally the same.



Now Mercury perceives a *Lorentz contraction* of its orbital path, and near its perihelion it goes faster than near its aphelion, so around the perihelion it perceives a shorter half-orbit than around the aphelion. Therefore it "thinks" it has not yet completed its perihelion half orbit and still has to travel some *distance*, whilst for a stationary observer it already completed it, thus yielding the *perihelion precession*.

Its *orbital path length* is Lorentz contracted by: $\frac{1}{\gamma} = \sqrt{1 - \frac{v^2}{c^2}}$ [15]

and the *radial* contraction must be by: $\sqrt{1 - \frac{2GM}{rc^2}} = \sqrt{1 - \frac{v_{esc}^2}{c^2}}$ [16]

From Kepler's 3rd law we obtain: $v = \sqrt{\frac{GM}{r}}$ [17]

and we also have: $v_{esc} = \sqrt{\frac{2GM}{r}}$ [18]

so there is a mismatch in the radial and orbital *length contraction*, such that $\sqrt{2}$ appears. That would of course be related to the curvature of spacetime as described by General Relativity, but for now I want to restrict myself to Special Relativity only. No tensor calculus. Keep it simple. Let's see what comes out if we apply "*Schwarzschild compression*" to Mercury's entire *distance* to the Sun and presume it experiences just Newtonian *gravitation*, but of course in its proper frame.

Legend:

With a prime: as perceived by Mercury itself,
without: as observed by a distant observer.

Contracted *radius*: $r' = r \cdot \sqrt{1 - \frac{2GM}{rc^2}} = \sqrt{r^2 - \frac{2GMr}{c^2}} = \sqrt{\frac{r^2c^2 - 2GMr}{c^2}} = \frac{\sqrt{r^2c^2 - 2GMr}}{c}$ [19]

orbital velocity: $v' = \sqrt{\frac{GM}{r'}} = \sqrt{\frac{GM}{\frac{\sqrt{r^2c^2-2GMr}}{c}}} = \sqrt{\frac{cGM}{\sqrt{r^2c^2-2GMr}}}$ [20]

$\beta' = \frac{v'}{c} = \sqrt{\frac{GM}{c \cdot \sqrt{r^2c^2-2GMr}}} = \sqrt{\frac{GM}{\sqrt{r^2c^4-2GMrc^2}}}$ [21]

Lorentz factor: $\frac{1}{\gamma'} = \sqrt{1-(\beta')^2} = \sqrt{1 - \frac{GM}{\sqrt{r^2c^4-2GMrc^2}}}$ [22]

we have: $\frac{GM}{\sqrt{r^2c^4-2GMrc^2}} = \frac{GM}{rc^2} + \left(\frac{GM}{rc^2}\right)^2 + \frac{3}{2}\left(\frac{GM}{rc^2}\right)^3 + \frac{5}{2}\left(\frac{GM}{rc^2}\right)^4 + \mathcal{O}\left(\left(\frac{GM}{rc^2}\right)^5\right)$ [23]

using [17]: $= \frac{v^2}{c^2} + \left(\frac{v^2}{c^2}\right)^2 + \frac{3}{2}\left(\frac{v^2}{c^2}\right)^3 + \frac{5}{2}\left(\frac{v^2}{c^2}\right)^4 + \mathcal{O}\left(\left(\frac{v^2}{c^2}\right)^5\right)$ [24]

hence: $(\beta')^2 = \beta^2 + \beta^4 + \frac{3}{2}\beta^6 + \frac{5}{2}\beta^8 + \mathcal{O}(\beta^{10})$ [25]

Bingo! A match in the lowest order, so we got rid of the aforementioned $\sqrt{2}$.

Ergo: applying *gravitational length contraction* to the entire *orbital radius* yields a consistent result.

A premise of SR is that both observers perceive the same mutual velocity, but v' (eq. [20], the Newtonian orbital *velocity* as perceived by Mercury itself) obviously exceeds v (Mercury's Newtonian *velocity* as expected by a distant observer). Even for circular orbits this yields a *perihelion precession*.

The *speed limit of light* forces $\beta' < 1$, yielding $\frac{r'}{r_s} > \frac{1}{2}$ or $\frac{r}{r_s} > \frac{1+\sqrt{2}}{2}$ which is $< \frac{3}{2}$. We found an ISCO (innermost stable circular orbit), but less than what is known as the photon sphere. Nevertheless, within an ISCO the "*centripetal acceleration*" required to compensate for the *centrifugal gravitational acceleration* cannot be achieved due to the *speed limit of light*, so the orbiting body has no other choice that to spiral inwards. We found it using only SR. See appendix I for some more maths about it.

We'll now calculate a very simple estimate of Mercury's *perihelion precession*. We calculate its *velocity* in the perihelion and aphelion and apply *inverse time dilation* to its *orbital period* using each of these *velocities*. Their difference divided by the *orbital period* yields a fraction, which would be an upper limit to the *perihelion precession* as an orbital fraction per orbit. We also estimate the average *distances* during both apside half orbits and estimate the *perihelion precession* using those. Therefore we add/subtract ¼ of the linear excentricity to/from the perihelion and aphelion distances. We use ¼ instead of ½ in an attempt to correct for the the fact that the apo-half lasts longer than the peri-half.

Next is a script written in JScript (javascript), which runs in the Windows Script Host (Cscript).

```
C:\Mercury> type mercury.js
var c   = 299792458; // speed of light [m/s]
var c2  = c*c;
var secondsPerMinute        = 60;
var minutesPerHour          = 60;
var hoursPerDay             = 24;
var daysPerJulianYear       = 365.25;
var yearsPerCentury         = 100;
var secondsPerHour          = secondsPerMinute     * minutesPerHour   ;
var secondsPerDay           = secondsPerHour       * hoursPerDay      ;
var secondsPerJulianYear    = secondsPerDay        * daysPerJulianYear;
var secondsPerJulianCentury = secondsPerJulianYear * yearsPerCentury  ;
var arcsecsPerArcmin = 60;
var arcminsPerDegree = 60;
var degreesPerCircle = 360;
var arcsecsPerDegree = arcsecsPerArcmin * arcminsPerDegree;
var arcsecsPerCircle = arcsecsPerDegree * arcsecsPerDegree;
// Mercury's orbit, from https://en.wikipedia.org/wiki/Mercury_(planet) :
var a   = 57909050e3;            // semimajor axis [m]
var e   = 0.205630;              // excentricity
var T   = 87.9691*secondsPerDay; // orbital period [s]
var orbitsPerJulianCentury  = secondsPerJulianCentury/T;
var arcsecsPerJulianCentury = arcsecsPerCircle*orbitsPerJulianCentury;
var c     = a*e;                // linear excentricity (focus-to-centre distance)
var b     = Math.sqrt(a*a-c*c); // semiminor axis
var A     = Math.PI*a*b;        // surface area of ellipse
var dA_dt = A/T;  // Kepler 2: dA/dt = 0.5*r*v is constant => v = 2*(dA/dt)/r
function gamma1(v) {return Math.sqrt(1-v*v/c2);}; // reciprocal Lorentz factor
function theta(vp,va)
{   var T1p  = T*gamma1(vp); // inverse time dilated orbital period using perihelion velocity
    var T1a  = T*gamma1(va); // inverse time dilated orbital period using aphelion velocity
    var dT   = T1a - T1p;    // their difference
    var dT_T = dT/T;         // as fraction of non-dilated period = orbital fraction per orbit
    return dT_T*arcsecsPerJulianCentury; // times "anglocity" => perihelion's anglocity
};
var rp = a-c, vp = 2*dA_dt/rp; // perihelion distance and velocity
var ra = a+c, va = 2*dA_dt/ra; //   aphelion distance and velocity
var ppUpper = theta(vp,va);    // based on velocities in perihelion and aphelion
rp += c/4; vp = 2*dA_dt/rp;    // approximate mean distance of half-orbit for both apsides
ra -= c/4; va = 2*dA_dt/ra;    // use 4 and not 2 because apo-half lasts longer
var ppMean = theta(vp,va);     // based on estimated mean velocities
WScript.StdOut.WriteLine("Mercury's perihelion precession, based on SR only:");
WScript.StdOut.WriteLine("  upper limit: " + ppUpper.toPrecision(2) + " arcsecs/century");
WScript.StdOut.WriteLine("    estimated: " + ppMean .toPrecision(2) + " arcsecs/century");

C:\Mercury> cscript //nologo mercury.js
Mercury's perihelion precession, based on SR only:
  upper limit: 59 arcsecs/century
    estimated: 43 arcsecs/century
```

The actual value is: 43 arcsecs/century.

Although the script merely calculates a rough estimate it feels like bingo once again.

If you cannot explain it simply, you do not understand it well enough.

Attributed to Albert Einstein, but he never said it.

Actually, he said to Louis de Broglie at the Gare du Nord in Paris
after they had visited the Fresnel centenary celebrations in 1927:

All physical theories, their mathematical expressions apart, ought to lend themselves to so simple a description that even a child could understand them.

Appendix I

ISCO by classical mechanics & special relativity only

Gravitational length contraction of entire *distance* to central *mass*:

$r' = r \cdot \sqrt{1 - \frac{r_S}{r}}$ where: $r_S = \frac{2GM}{c^2}$

substitute: $x = \frac{r}{r_S}$ then: $x' = x \cdot \sqrt{1 - \frac{1}{x}}$

Newtonian gravitation: $F_g = \frac{GMm}{(r')^2} = \mu(\omega')^2 r'$ where: $\mu = \frac{Mm}{M+m}$ (*reduced mass*)

hence: $\frac{GMm}{(r')^2} = \frac{Mm(\omega')^2 r'}{M+m}$ so: $\frac{G}{(r')^2} = \frac{(\omega')^2 r'}{M+m}$

which yields: $\frac{GM_{tot}}{r'} = (\omega')^2 (r')^2$

Speed limit of light: $v = \omega r < c$ as well as: $v' = \omega' r' < c$

therefore: $\frac{GM_{tot}}{r'} = (\omega')^2 (r')^2 < c^2$ $\therefore \ \frac{1}{r'} < \frac{c^2}{GM_{tot}}$ $\therefore \ r' > \frac{GM_{tot}}{c^2} = \frac{r_{S,M_{tot}}}{2}$

yielding: $x' > \frac{1}{2}$ (setting $\beta' = 1$ in [21] yields the same).

This suggests we should always use M_{tot} for *gravitational length contraction*. x' is the contracted *distance* to M in terms of the (nonconctracted) *Schwarzschild radius*. At $x' = \frac{1}{2}$ the proper *orbital velocity* would equal the *speed of light*. For $x' < \frac{1}{2}$ the orbiting body MUST spiral inwards. For the noncontracted *distance* to M we find:

as stated above: $x' = x \cdot \sqrt{1 - \frac{1}{x}}$ $\therefore \ (x')^2 = x^2 \cdot \left(1 - \frac{1}{x}\right) = x^2 - x$

$\therefore \ x^2 - x - (x')^2 = 0$ $\therefore \ x = \frac{1 \pm \sqrt{(-1)^2 - 4 \cdot 1 \cdot (-(x')^2)}}{2} = \frac{1 \pm \sqrt{1 + 4(x')^2}}{2}$

outside r_S we have $x > 1$: choose the plus sign: $x = \frac{1 + \sqrt{1 + 4(x')^2}}{2}$

which for $x' = \frac{1}{2}$ yields: $x = \frac{r}{r_S} = \frac{1 + \sqrt{1 + 4\left(\frac{1}{2}\right)^2}}{2} = \frac{1 + \sqrt{2}}{2}$

for $m \ll M$ we get: $M_{tot} \approx M$ $\therefore \ r_S \approx r_{S,M_{tot}}$

Using only classical mechanics and special relativity (and presuming we should always use M_{tot} for *gravitational length contraction*) we found an ISCO (as perceived by a distant observer) equal to:

$$\frac{1 + \sqrt{2}}{2} \cdot r_{S,M_{tot}} \approx 1.207\,106\,781\,186\,5474 \cdot r_{S,M_{tot}}$$

This is however less than what is given on https://en.wikipedia.org/wiki/Schwarzschild_geodesics :

$$r_{\text{inner}} \approx \frac{3}{2} r_S$$

yielding the photon sphere at $x = \frac{3}{2}$ and an ISCO at $x = 3$. The derivation given there uses r (*radius* as seen by distant observer) and τ (proper time of orbiting body), thereby intermixing the frames (which doesn't feel good to me). It also uses the *angular momentum*, which the above does not. But I am not saying $x = \frac{3}{2}$ is wrong!

