Dimensionless distance & time: $\rho := r/r_S\,,\quad \tau := ct/r_S$

Newsteinian free fall: $E_k = (\gamma-1)mc^2 = GMm/r = -E_p$

$\gamma - 1 = GM/rc^2$

$\gamma = 1 + \frac{r_S}{2r} = 1 + \frac{1}{2\rho}$

Lorentz factor: $\boldsymbol{\gamma = \frac{2\rho+1}{2\rho}}$ $\qquad \rho = 1 \Rightarrow \gamma = \frac{3}{2}$

Inverse of Lorentz factor: $\boldsymbol{\rho = \frac{1}{2(\gamma-1)}}$

$\gamma = 1/\sqrt{1-\beta^2} \therefore \beta = \sqrt{1-1/\gamma^2}$

Velocity (positive = outgoing): $\boldsymbol{\beta = \frac{\sqrt{4\rho+1}}{2\rho+1}} = \frac{d\rho}{d\tau}$ $\qquad \rho = 1 \Rightarrow \beta = \frac{\sqrt{5}}{3}$

Inverse of velocity: $\boldsymbol{\rho = \frac{1-\beta^2+\sqrt{1-\beta^2}}{2\beta^2}}$

differential time: $d\tau = \frac{d\rho}{\beta} = \frac{2\rho+1}{\sqrt{4\rho+1}}d\rho$

Time interval (growing for outgoing): $\boldsymbol{\Delta\tau = \frac{1}{3}(\rho+1)\sqrt{4\rho+1}}\ \Big]_{\rho_0}^{\rho_1}$

$\rho = 1 \Rightarrow \tau = \frac{2\sqrt{5}}{3}$

$\rho = 0 \Rightarrow \tau = \frac{1}{3}$

Inverse function of time: $\boldsymbol{\rho}$ = something nasty[1]

Acceleration = 1st fluction of velocity: $\alpha = \frac{d\beta}{d\tau} = \frac{d\beta}{d\rho}\cdot\frac{d\rho}{d\tau} = \frac{d\beta}{d\rho}\beta$

we find: $\frac{d\beta}{d\rho} = \frac{-4\rho}{(2\rho+1)^2\sqrt{4\rho+1}}$

hence: $\alpha = \frac{-4\rho}{(2\rho+1)^2\sqrt{4\rho+1}}\cdot\frac{\sqrt{4\rho+1}}{2\rho+1}$

rendering: $\boldsymbol{\alpha = \frac{-4\rho}{(2\rho+1)^3}}$

Inverse of acceleration: $\boldsymbol{\rho}$ = yet something nasty[2]

Relativistic momentum: $p = \frac{m_{rest}\beta c}{\sqrt{1-\beta^2}}$

dimensionless specific momentum: $\boldsymbol{\tilde{p} = \frac{\beta}{\sqrt{1-\beta^2}} = \beta\gamma = \frac{\sqrt{4\rho+1}}{2\rho}}$

Dimensionless force = 1st fluction thereof: $\tilde{\mathcal{F}} = \dot{\tilde{p}} = \frac{d\tilde{p}}{d\beta}\cdot\frac{d\beta}{d\tau} = \frac{d\tilde{p}}{d\beta}\alpha$

i.e.: $\tilde{\mathcal{F}} = \frac{1}{(1-\beta^2)^{3/2}}\cdot\frac{-4\rho}{(2\rho+1)^3}$

or: $\tilde{\mathcal{F}} = \frac{1}{\left(1-\frac{4\rho+1}{(2\rho+1)^2}\right)^{3/2}}\cdot\frac{-4\rho}{(2\rho+1)^3}$

if $\rho > 0$: $\boldsymbol{\tilde{\mathcal{F}} = \frac{-1}{2\rho^2}}$ (brakes outward speed)

which translates back to: $\frac{d\tilde{p}}{d\tau} = \frac{d\left(\frac{p}{m_{rest}c}\right)}{d\left(\frac{ct}{r_S}\right)} = \frac{\frac{1}{m_{rest}c}}{\frac{c}{r_S}}\cdot\frac{dp}{dt} = \frac{r_S}{m_{rest}c^2}\dot{p} = \frac{-r_S^2}{2r^2}$

or: $\boldsymbol{F} = \frac{-r_S m_{rest}c^2}{2r^2} = \frac{-\frac{2GM}{c^2}m_{rest}c^2}{2r^2} = \boldsymbol{\frac{-GMm_{rest}}{r^2}}$

which is Newton's law of gravitation, so all seems consistent.

With the inverse Lorentz factor, we get: $\boldsymbol{\tilde{\mathcal{F}} = \frac{r_S}{m_{rest}c^2}\dot{p} = \frac{1}{8(\gamma-1)^2} \therefore F = \frac{m_{rest}c^2}{8r_S(\gamma-1)^2}}$

(don't know how useful this last eqn. might be).

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[1] https://www.wolframalpha.com/input?i=inverse+function+%28x%2B1%29%E2%88%9A%284x%2B1%29%2F3

[2] https://www.wolframalpha.com/input?i=inverse+function+%28-4x%29%2F%282x%2B1%29%5E3

Perceived by falling victim:

contracted distance: $\boldsymbol{\varrho = \frac{\rho}{\gamma} = \frac{2\rho^2}{2\rho+1}}$

inverse thereof: $\boldsymbol{\rho = \frac{\varrho+\sqrt{\varrho(\varrho+2)}}{2}}$

Lorentz factor: $\boldsymbol{\gamma' = \frac{1+\varrho+\sqrt{\varrho(\varrho+2)}}{\varrho+\sqrt{\varrho(\varrho+2)}}}$

Velocity: $\boldsymbol{\beta' = \frac{\sqrt{1+4\rho}}{1+2\rho} = \frac{\sqrt{1+2\varrho+2\sqrt{\varrho(\varrho+2)}}}{1+\varrho+\sqrt{\varrho(\varrho+2)}} = \frac{d\varrho}{d\tau'}}$

> These β & γ differ from http://henk-reints.nl/astro/HR-fall-into-black-hole-slides.pdf under "Various free fall velocities to point mass", where $\gamma(\varrho)$ is based on the contracted potential. Here I simply substituted $\rho(\varrho)$, which seems incorrect, so next derivations are probably flawed as well, but for now, I'll leave it as is.

differential proper time: $d\tau' = \frac{d\varrho}{\beta'} = \frac{1+\varrho+\sqrt{\varrho(\varrho+2)}}{\sqrt{1+2\varrho+2\sqrt{\varrho(\varrho+2)}}}d\varrho$

Proper time (omitting integr. const.): $\boldsymbol{\tau' =}$

$$\frac{(\varrho+2)\sqrt{2\varrho+2\sqrt{\varrho(\varrho+2)}+1}\left(16\varrho^5+8\left(2\sqrt{\varrho(\varrho+2)}+9\right)\varrho^4+4\left(14\sqrt{\varrho(\varrho+2)}+27\right)\varrho^3+60\left(\sqrt{\varrho(\varrho+2)}+1\right)\varrho^2+\left(20\sqrt{\varrho(\varrho+2)}+9\right)\varrho+\sqrt{\varrho(\varrho+2)}\right)}{6\left(1+\varrho+\sqrt{\varrho(\varrho+2)}\right)^2\left(2\varrho^3+2\left(\sqrt{\varrho(\varrho+2)}+3\right)\varrho^2+4\left(\sqrt{\varrho(\varrho+2)}+1\right)\varrho+\sqrt{\varrho(\varrho+2)}\right)}$$

$$+\arctan\frac{1}{\sqrt{2\varrho+2\sqrt{\varrho(\varrho+2)}+1}}$$

Well done, WolframAlpha! (hopefully I made no errors retyping it)

$$\rho = 0 \Longrightarrow \tau' = \frac{1}{3}+\frac{\pi}{4}$$

Acceleration: $\boldsymbol{\alpha' =} \frac{d\beta'}{d\tau'} = \frac{d\beta'}{d\varrho}\cdot\frac{d\varrho}{d\tau'} = \frac{d\beta'}{d\varrho}\beta'$

$$= \frac{d}{d\varrho}\left(\frac{\sqrt{1+2\varrho+2\sqrt{\varrho(\varrho+2)}}}{1+\varrho+\sqrt{\varrho(\varrho+2)}}\right)\cdot\frac{\sqrt{1+2\varrho+2\sqrt{\varrho(\varrho+2)}}}{1+\varrho+\sqrt{\varrho(\varrho+2)}}$$

$$= \frac{-\varrho-\sqrt{\varrho(\varrho+2)}}{\sqrt{\varrho(\varrho+2)}\left(1+\varrho+\sqrt{\varrho(\varrho+2)}\right)\sqrt{1+2\varrho+2\sqrt{\varrho(\varrho+2)}}}\cdot\frac{\sqrt{1+2\varrho+2\sqrt{\varrho(\varrho+2)}}}{1+\varrho+\sqrt{\varrho(\varrho+2)}}$$

$$\boldsymbol{\alpha' = \frac{-\varrho-\sqrt{\varrho(\varrho+2)}}{\sqrt{\varrho(\varrho+2)}\left(1+\varrho+\sqrt{\varrho(\varrho+2)}\right)^2}}$$

Momentum: $\boldsymbol{\tilde{p}' = \frac{\beta'}{\sqrt{1-(\beta')^2}} = \beta'\gamma' = \frac{\sqrt{1+2\varrho+2\sqrt{\varrho(\varrho+2)}}}{\varrho+\sqrt{\varrho(\varrho+2)}}}$

Force: $\tilde{\mathcal{F}}' = \frac{d\tilde{p}'}{d\tau'} = \frac{d\tilde{p}'}{d\beta'}\frac{d\beta'}{d\tau'} = \frac{d\tilde{p}'}{d\beta'}\alpha'$

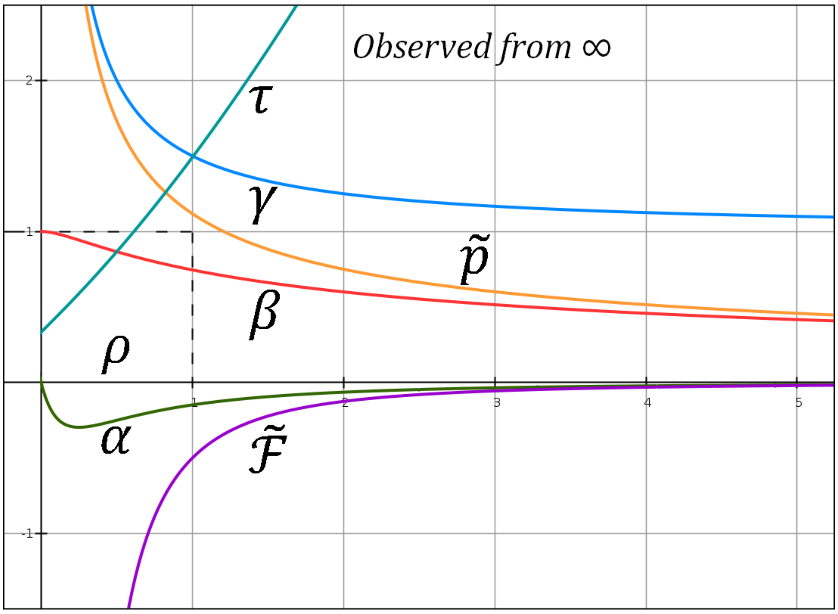
i.e.: $\tilde{\mathcal{F}}' = \frac{1}{(1-(\beta')^2)^{3/2}}\cdot\frac{-\varrho-\sqrt{\varrho(\varrho+2)}}{\sqrt{\varrho(\varrho+2)}\left(1+\varrho+\sqrt{\varrho(\varrho+2)}\right)^2}$

or: $\tilde{\mathcal{F}}' = \frac{1}{\left(1-\frac{1+2\varrho+2\sqrt{\varrho(\varrho+2)}}{\left(1+\varrho+\sqrt{\varrho(\varrho+2)}\right)^2}\right)^{3/2}}\cdot\frac{-\varrho-\sqrt{\varrho(\varrho+2)}}{\sqrt{\varrho(\varrho+2)}\left(1+\varrho+\sqrt{\varrho(\varrho+2)}\right)^2}$

if $\rho > 0$: $\boldsymbol{\tilde{\mathcal{F}}' = \frac{-\sqrt{1+\varrho+\sqrt{\varrho(\varrho+2)}}}{\varrho\sqrt{2\varrho(\varrho+2)}\left(\sqrt{\varrho}+\sqrt{\varrho+2}\right)}}$

Laurent series $@\varrho = \infty$: $\boldsymbol{\tilde{\mathcal{F}}' = \frac{-1}{2\varrho^2}} + \frac{1}{2\varrho^3} - \frac{3}{4\varrho^4} + \frac{5}{4\varrho^5} + \mathcal{O}\left(\frac{1}{\varrho^{11/2}}\right)$

1st term = Newtonian.



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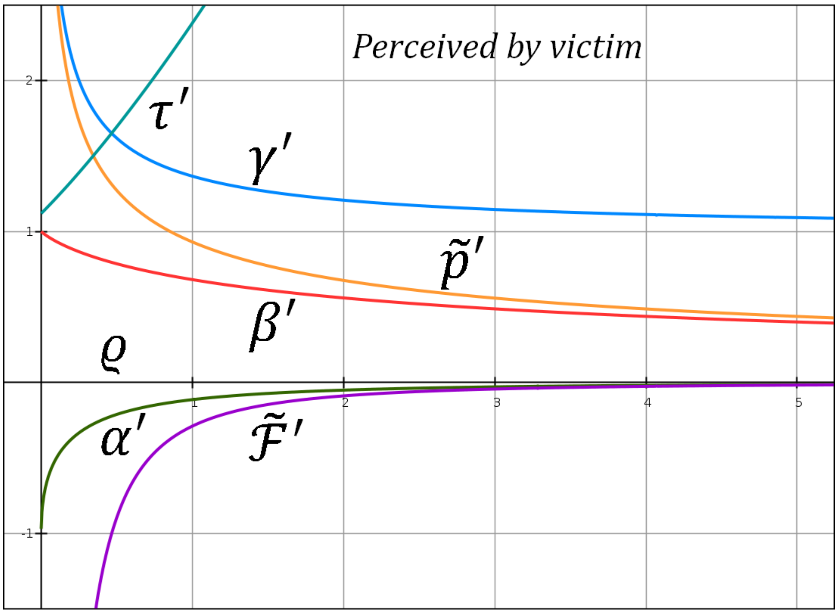

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Spaghettified by a sadistic photographahahahah...