

Potential energy: $E_p = \frac{-GMm}{r}$

Circular orbit: $\frac{v_{\text{orb}}^2}{r} = g = \frac{GM}{r^2}$

Orbital velocity: $v_{\text{orb}} = \sqrt{\frac{GM}{r}}$

Orbital kinetic energy: $E_{k,\text{orb}} = \frac{mv_{\text{orb}}^2}{2} = \frac{GMm}{2r}$

Total energy (Hamiltonian): $H = E_{k,\text{orb}} + E_p = \frac{-GMm}{2r}$

Angular momentum: $L = mv_{\text{orb}}r = m\sqrt{GMr}$

Dimension (Mass, Distance, Time): $[E] = [M][D^2][T^{-2}]$
and: $[L] = [M][D^2][T^{-1}] = [E][T] \therefore [L][E^{-1}] = [T]$

Circular orbit: $\frac{L}{H} = \frac{m\sqrt{GMr}}{-GMm/2r} = \frac{-2r\sqrt{GMr}}{GM} = \frac{-2r\sqrt{r}}{\sqrt{GM}} = \frac{-2\pi r}{\pi v_{\text{orb}}} = \frac{-\Delta t_{\text{orb}}}{\pi}$

i.e.: $\Delta t_{\text{orb}} = -\pi L/H$

PRESUMPTION: this also applies to non-circular orbits (I allow YOU to prove it¹).

We have: $L = mvr \sin(\theta)$

where: $\theta =$ angle of approach, such that zero = bull's eye

then: $\Delta t_{\text{orb}} = -\pi vr \sin(\theta) \cdot m/H$

i.e.: $\Delta t_{\text{orb}} = -\pi vr \sin(\theta) / \tilde{H}$

where: $\tilde{H} = H/m$ (specific Hamiltonian)

Hamiltonian in general: $\tilde{H} = \tilde{E}_k + \tilde{E}_p = \frac{v^2}{2} - \frac{GM}{r} = \frac{rv^2 - 2GM}{2r}$

Suppose we measured: $P_1 = (t_1, r_1, \varphi_1) \quad \therefore x_1 = r_1 \cos \varphi_1, y_1 = r_1 \sin \varphi_1$
 $P_2 = (t_2, r_2, \varphi_2) \quad \therefore x_2 = r_2 \cos \varphi_2, y_2 = r_2 \sin \varphi_2$
 where both P_1 & P_2 are in Q_1

estimate: $v^2 = \frac{(x_2 - x_1)^2 + (y_2 - y_1)^2}{(t_2 - t_1)^2}$

and: $r = \frac{r_1 + r_2}{2}$

as well as: $\theta = \frac{\varphi_1 + \varphi_2}{2} + \text{atan2}(y_2 - y_1, x_1 - x_2)$

yes, indexes are correct: 2nd y minus 1st & 1st x minus 2nd

Then we know: $\tilde{H} = \frac{rv^2 - 2GM}{2r}$

as well as: $\tilde{L} = vr \sin(\theta)$

and, last but not least: $\Delta t_{\text{orb}} = \frac{-\pi vr \sin(\theta)}{\tilde{H}} = \frac{2\pi vr^2 \sin(\theta)}{2GM - rv^2} = \frac{2\pi r \tilde{L}}{2GM - rv^2}$

Merely two positions & their intermediate time span would suffice to **estimate** entire orbit!

This Δt_{orb} has an asymptote (restricting closed orbits; for hyperbolic paths, it yields a (negative) time for which I have not (yet) investigated to which points of the trajectory it relates)

at: $rv^2 = 2GM$

it obviously requires: $rv^2 < 2GM$ (presuming $\sin(\theta) > 0$)

i.e.: $r \frac{v^2}{c^2} < \frac{2GM}{c^2} \therefore r\beta^2 < r_s \therefore \frac{r}{r_s} \beta^2 = \rho\beta^2 < 1$

or: $\beta^2 < 1/\rho =$ dimensionless proximity.

Outside Schwarzschild radius (i.e. $\rho > 1$), Newtonian mechanics enforces speed limit of light on closed orbits.

¹ See: <http://henk-reints.nl/astro/HR-orbital-period.pdf>