

$$\text{Mass of homogeneous sphere: } M = \rho V = \frac{4}{3}\pi\rho r^3 \quad [1]$$

$$\text{Schwarzschild radius: } r_S = \frac{2GM}{c^2} \therefore \frac{r_S}{r} = \frac{2GM}{rc^2} = \frac{2G}{rc^2} \cdot \frac{4}{3}\pi\rho r^3 = \frac{8\pi G\rho}{3c^2} r^2 \quad [2]$$

$$\text{Escape velocity: } v_{esc} = \sqrt{\frac{2GM}{r}} \therefore v_{esc}^2 = \frac{2GM}{r} = \frac{r_S c^2}{r} \therefore \frac{r_S}{r} = \frac{v_{esc}^2}{c^2} \quad [3]$$

$$\text{Lorentz factor: } \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad [4]$$

$$\text{Relativistic mass: } m = \gamma m_0 \quad [5]$$

$$\text{Kinetic energy: } E_k = mc^2 - m_0 c^2 = \gamma m_0 c^2 - m_0 c^2 = (\gamma - 1)m_0 c^2 \quad [6]$$

$$\therefore \frac{E_k}{m_0 c^2} = \gamma - 1 \therefore 1 + \frac{E_k}{m_0 c^2} = \gamma \therefore \gamma = \frac{m_0 c^2 + E_k}{m_0 c^2} \quad [7]$$

$$\text{substitute [4]: } \therefore \sqrt{1 - \frac{v^2}{c^2}} = \frac{m_0 c^2}{m_0 c^2 + E_k} \quad [8]$$

$$\text{Escape energy vs. radius: } \sqrt{1 - \frac{v_{esc}^2}{c^2}} = \frac{m_0 c^2}{m_0 c^2 + E_{esc}} \quad [9]$$

$$\text{using [3]: } \therefore \sqrt{1 - \frac{r_S}{r_{esc}}} = \frac{m_0 c^2}{m_0 c^2 + E_{esc}} \therefore 1 - \frac{r_S}{r_{esc}} = \left(\frac{m_0 c^2}{m_0 c^2 + E_{esc}} \right)^2 \quad [10]$$

$$\therefore \frac{r_S}{r_{esc}} = 1 - \left(\frac{m_0 c^2}{m_0 c^2 + E_{esc}} \right)^2 \quad [11]$$

$$\text{from [2]: } \frac{r_S}{r_{esc}} = \frac{8\pi G\rho}{3c^2} r_{esc}^2 \quad [12]$$

$$\therefore r_{esc}^2 = \frac{3c^2}{8\pi G\rho} \cdot \left[1 - \left(\frac{m_0 c^2}{m_0 c^2 + E_{esc}} \right)^2 \right] \quad [13]$$

$$\therefore r_{esc} = c \cdot \sqrt{\frac{3}{8\pi G\rho}} \cdot \sqrt{1 - \left(\frac{m_0 c^2}{m_0 c^2 + E_{esc}} \right)^2} \quad [14]$$

An object with rest mass m_0 and *kinetic energy* E_{esc} can/will escape if it is at a *distance* of at least r_{esc} .

$$\text{For a neutron star, we've got: } \rho = \rho_N = 1.392134 \times 10^{18} \text{ kg/m}^3 \quad [15]$$

$$\text{and of course: } c = 299792458 \text{ m/s} \quad [16]$$

$$\text{as well as: } G = 6.67408 \times 10^{-11} \text{ m}^3/\text{kg/s}^2 \quad [17]$$

$$\text{so: } c \cdot \sqrt{\frac{3}{8\pi G\rho}} = 299792458 \cdot \sqrt{\frac{3}{8\pi \cdot 6.67408 \times 10^{-11} \cdot 1.392134 \times 10^{18}}} = 10745 \text{ m} \quad [18]$$

This value equals the *critical black hole radius* as calculated in <http://henk-reints.nl/astro/HR-on-the-universe.php>.

Suppose this object is an electron originating from a decaying neutron of a neutron star without obtaining *energy* from the star's *binding energy*. Then:

$$\text{Electron mass: } m_0 = m_e = 9.10938356 \times 10^{-31} \text{ kg} \quad [19]$$

$$\text{so: } m_0 c^2 = m_e c^2 = 8.18710565 \times 10^{-14} \text{ J} \quad [20]$$

$$\text{Neutron decay energy: } E_{dec} = (m_n - m_p - m_e)c^2 = 0.78257 \text{ MeV} = 1.2538 \times 10^{-13} \text{ J} \quad [21]$$

First assume the electron obtains all this *energy*. Then:

$$r_0 = 10745 \cdot \sqrt{1 - \left(\frac{8.18710565 \times 10^{-14}}{8.18710565 \times 10^{-14} + 1.2538 \times 10^{-13}} \right)^2} = 9871 \text{ m} \quad [22]$$

$$\text{Its corresponding mass equals: } M_0 = \frac{4}{3}\pi\rho r_0^3 = 7.234 \times 10^{30} \text{ kg} \approx 3.637 M_\odot \quad [23]$$

So a neutron star with a diameter that exceeds 19.74 km and thus a *mass* above 3.637 times that of our sun cannot lose any electrons by neutron decay because *gravitation* definitely wins that game. Above this size a ball of neutronium would not be able to disintegrate. That's why I do still not really understand what the big bang actually was. The *IniAll* must have had a *radius* of nearly 3 astronomical units.

In practice, the electron antineutrino takes away roughly 50% of the neutron decay *energy* on average,

yielding:
$$r_1 = 10745 \cdot \sqrt{1 - \left(\frac{8.18710565 \times 10^{-14}}{8.18710565 \times 10^{-14} + \frac{1.2538 \times 10^{-13}}{2}} \right)^2} = 8856 \text{ m} \quad [24]$$

corresponding to a *diameter* of:
$$d_1 = 17.71 \text{ km} \quad [25]$$

and its corresponding *mass* is:
$$M_1 = \frac{4}{3} \pi \rho r_1^3 = 4.050 \times 10^{30} \text{ kg} \approx 2.036 M_\odot \quad [26]$$

To me it is remarkable that this *mass* is within the tolerance of the lower bound of the TOV-limit as stated on https://en.wikipedia.org/wiki/Tolman%E2%80%93Oppenheimer%E2%80%93Volkoff_limit#History (as of 2019-08-27): The mass of the pulsar PSR J0348+0432, at 2.01 ± 0.04 solar masses, puts an empirical lower bound on the TOV limit.

Below this *mass*, a neutron star would be prone to evaporation by neutron decay. This could mean twice the solar *mass* is a lower *mass* limit for neutron stars to be able to stay alive at all. But the neutrons in a neutron star are definitely not free neutrons, so they may have a far longer *decay time*, which would also avoid this evaporation.