The triplets paradox

Please also read http://henk-reints.nl/astro/HR-before-the-big-bang.pdf, it describes the twin paradox a little bit more detailed than below and it gives some more insight in *time dilation*.

Twin paradox

In brief, Tyler and Sophia are twins who are separated at birth. Sophia leaves at a high *velocity* with a *Lorentz factor* of say $\gamma = 2$ (relative to Tyler), who stays home. *Time dilation* then implies that Tyler sees Sophia age at half *speed*. After 20 years of Tyler *time*, she returns home, having travelled at the same *velocity* during the entire journey. Due to this *time dilation*, she'll be just 10, whilst Tyler has become 20. But from Sophia 's perspective, Tyler has travelled all the *time* at this *velocity*, so according to her, *time dilation* affected him, and then, since she's 10 (it's her *time* itself that ran at half *speed* as seen by Tyler), he's merely 5 years old when they meet again. Huh?

The solution is that only Sophia is affected by *gravitational time dilation* during the *accelerations* she undergoes at departure (speeding up), turning around (decelerating and "recelerating"), and returning home (braking), which causes the asymmetry between them. On return, Sophia is indeed 10 years old and Tyler's *age* is 20.

Triplets

But now suppose Sophia has got an identical twin sister named Sophib. Then we've got a two-egg triplet. In all regards (except the last letter of their names), Sophib is identical to Sophia, which includes their *masses* and every other relevant property. And Sophib goes travelling as well, but in the exact opposite direction. All her movements relative to Tyler are identical to, and (in Tyler *time*) synchronised with what Sophia does, but mirrored. Then the very same applies to Tyler & Sophib as what's just described for Tyler & Sophia.

Both Sophia and Sophib will have reached the *age* of 10 when they return home and Tyler, who never underwent any *gravitational time dilation*, is 20. *Gravitational time dilation applies to* Sophi* only. That's why they are really 10 on arrival, and Tyler has become their older triplet brother, born the same day, but aged 20.

Here come the girls...

Now we'll look at the girls only! Well, we'll still need Tyler, but only in order to calculate the girls's mutual *velocity*, which can easily be calculated.

We've got the *Lorentz factor*: $\gamma = \frac{1}{\sqrt{1-\beta^2}} \therefore \beta = \sqrt{1 - \frac{1}{\gamma^2}}$

And the relativistic superposition (not: addition!) of two *velocities* along a straight line:

$$\beta_{12} = \frac{\beta_1 + \beta_2}{1 + \beta_1 \beta_2}$$

From Sophia's perspective, we must superimpose Sophib's *velocity* relative to Tyler on Tyler's *velocity* with respect to to her. Both *velocities* have a *Lorentz factor* of $\gamma = 2$. From Sophib's perspective it is likewise, we'll just need to exchange the "a" and "b" in the girls' names.

So we've got: $\gamma_1 = \gamma_2 = 2 \therefore \beta_1 = \beta_2 = \sqrt{1 - \frac{1}{2^2}} = \frac{1}{2}\sqrt{3}$ (≈ 0.8660) relative to Tyler

and: $\beta_{12} = \frac{\frac{1}{2}\sqrt{3} + \frac{1}{2}\sqrt{3}}{1 + \frac{1}{2}\sqrt{3} \cdot \frac{1}{2}\sqrt{3}} = \frac{\sqrt{3}}{1 + \frac{3}{4}} = \frac{4}{7}\sqrt{3}$ (≈ 0.9897) relative to one another

therefore: $\gamma_{12} = \frac{1}{\sqrt{1 - \left(\frac{4}{7} \cdot \sqrt{3}\right)^2}} = \frac{1}{\sqrt{1 - \frac{16}{49} \cdot 3}} = \frac{1}{\sqrt{1 - \frac{48}{49}}} = 7$

We could also have done some simple and easy freshling algebra:

$$\gamma_{12} = \frac{1}{\sqrt{1-\beta_{12}^2}} = \frac{1}{\sqrt{1-\left(\frac{\beta_1+\beta_2}{1+\beta_1\beta_2}\right)^2}} = \frac{1}{\sqrt{1-\left(\frac{\sqrt{1-1/\gamma_1^2}+\sqrt{1-1/\gamma_2^2}}{1+\sqrt{1-1/\gamma_1^2}\cdot\sqrt{1-1/\gamma_2^2}}\right)^2}} \quad = \quad \text{O.M.G.}$$

Please feel challenged and do this exercise yourself! If you work it out correctly, it should finally yield:

$$\gamma_{12} = \gamma_1\gamma_2 + \sqrt{(\gamma_1^2-1)(\gamma_2^2-1)}$$

If γ_1 and γ_2 do not differ too much or are large,
this is approximated by: $\gamma_{12} \approx 2\gamma_1\gamma_2 - 1$

and if they are equal it becomes: $\gamma_{12} = 2\gamma_1^2 - 1$ which is exact,

so for Sophi* we obtain: $\gamma_1 = \gamma_2 = 2 \therefore \gamma_{12} = 2\cdot 2^2 - 1 = 7$

which of course is the same as what was already found. In case you consider these nice formulas you can reuse, then please note they apply only to *velocities* along a straight line, i.e. with a zero *angle* between them.

This *Lorentz factor* of $\gamma = 7$ is to be used for the girls' mutual *time dilation*, which leads us to the...

Twin paradox once again

From Sophia's perspective, Sophib would be just $\frac{10}{7} \approx 1.43$ years old when they meet again, and from Sophib's perspective Sophia would be just 1.43 years old. But according to Tyler, who's been awaiting them for 20 years, both are 10. And according to themselves, each of them is 10 as well. But both Sophia and Sophib see their little sister being a mere 1.43 years old... Since they have undergone exactly the very same and identical *gravitational time dilation*, it can in no way remove this paradox.

We can even completely leave out Tyler and just look at Sophia and Sophib, being - apart from all they need for the experiment - the only two entities in a further empty space, performing a perfectly symmetrical maneuver, accelerating away from each other for a while, then travelling at a constant *speed* for a bit less than 5 years (each measures it in her own *time*), then decelerating and "recelerating" towards each other, travelling at a constant *speed* once again, and finally decelerating, thus avoiding a hard collision, enabling them to simply kiss one another. Both have an identical *mass*, and they feel exactly the same *forces* exerted on them by their engines, so all *accelerations* are identical but in the opposite direction. Instead of using engines (they would also need to be perfectly identical), they can also connect themselves to one another with a long enough massless and unbreakable rope having a spring at its centre, which, thanks to Hooke's law (ceiiinosssttuu[1]), will eventually bring the girls back together (to get her?), 10 years after they perfectly symmetrically (Newton's 3rd law) pushed each other away resulting in $\gamma = 7$. If they're able to push that hard, they can also cope with the final collision...

Everything will be perfectly symmetrical all the way. So on balance, in this situation there's only *special relativistic time dilation*, which is identical in each observer's own local frame. Both watch their sister aging very slowly at just $\frac{1}{7}$ of their own pace. At the end of their respective journeys both are 10 as measured in their own *time*, and both 10 year old girls will greet the other by saying: "*Hello, baby!*" to their twin sister that's merely 1.43 years old, still wearing a nappy (or a diaper for them yanks[2] who ain't gotten the rite grammer... ☺).

Have you got a proper solution to this paradox?

Contemplations

Of course I have contemplated it. First of all, I can think of no reason at all why it would be fundamentally impossible to perform such an experiment. There must exist a solution. But there could somewhere be some

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[1] Robert Hooke: "A description of helioscopes, and some other instruments", 1676, p.31; and: "De potentia restitutiva", 1678, p.1, where he reveals the anagram as: Ut tensio sic vis.

[2] Did you know the term yankees originates from the Dutch names Jan & Kees? These names in turn originate from Johannes & Cornelis (John & Neil) Tip: use the speaker icon in [Google Translate]() in order to here the proper Dutch pronunciation).

hidden discontinuity in one quantity or the other, which would give rise to some undetermined value. The event of the observed clock showing zero (i.e. t_0') might then be anywhere relative to t_0 in the observer's *time*, which would solve it.

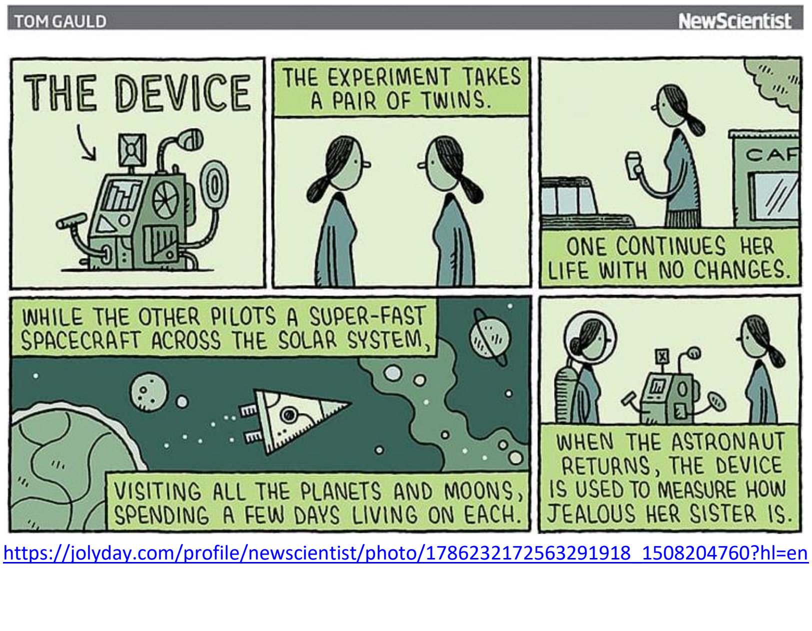
In the original twin paradox, Sophia's departure, turn around, and return are abrupt *velocity changes* in litterally no time. So there we got it: infinite *acceleration* causing infinite *gravitational time dilation*, which "inflates" this zero *duration* to $0 \times \infty = ?$ which can correctly place t_0' relative to t_0.

But in a more realistic scenario there can only be a finite *acceleration*. But that itself might suddenly start and stop if the exerted *force* has abrupt changes. Then once again we've got a discontinuity, although I do not yet see how a discontinuous *force* on a given *mass* would yield an undetermined t_0'. Must $\frac{\partial a}{\partial t}$ also be continuous? Or maybe even all $\frac{\partial^n}{\partial t^n} v(t)$ up to some large value of n? I happen to be victim of the *use it or lose it* principle as far as some mathematical skills like integration and tensor calculus are concerned by not doing that for over 35 years, so I'm no longer able to quickly and easily dive very deep into this. But I think it must be some type of discontinuity that can put t_0' anywhere relative to t_0.

I have not any reason to doubt Einstein's ~~theory~~ conclusion of relativity. Both postulates (the principle of relativity and the principle of the constancy of the *speed of light*) as well as the equivalence principle are not just excogitations, they are concluded from observed phenomena. The 2nd postulate isn't even a postulate since it is already contained in Maxwell's equations, as Einstein himself already wrote in footnote 2 of his $E = mc^2$ article, where he had already dropped this postulate even before his original paper had been published.

> $c = 1/\sqrt{\varepsilon_0 \mu_0}$, where both ε_0 and μ_0 are properties of empty space. Not any observation contradicts their constancy. Since empty space has no reference points at all, it cannot have any *velocity* relative to whatever, not even zero. Therefore ε_0 and μ_0 cannot depend on any mutual *velocity* between two observers, nor can the *speed of light*. Q.E.D.

This leaves his 1st postulate, the principle of relativity: in any local frame the same laws of physics apply, independent of the *velocity* of any other object (a fact of experience that should be familiar to everyone), and the equivalence principle: inertia is the same as gravitation (if blindfolded you cannot distinguish between *acceleration* and *gravity*). Facts. Observed phenomena. No fabrications. In "probably the 2nd or 3rd week of October 1907" Einstein realised that a free falling man does not feel his own weight. He was the very first who conceived the concept of weightlessness. Free fall is equivalent to inertial movement according to Newton's 1st law, the law of inertia.



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