Ball thrown through train in driving direction.

$tp =$ train w.r.t. platform
$bt =$ ball w.r.t. train
$bp =$ ball w.r.t. platform

Velocities etc.:

$$\beta_{tp} \qquad \gamma_{tp}^2 = \frac{1}{1-\beta_{tp}^2}$$

$$\beta_{bt} \qquad \gamma_{bt}^2 = \frac{1}{1-\beta_{bt}^2}$$

$$\beta_{bp} = \frac{\beta_{tp}+\beta_{bt}}{1+\beta_{tp}\beta_{bt}} \qquad \gamma_{bp}^2 = \frac{1}{1-\beta_{bp}^2}$$

In general:

$$\beta^2 = \frac{\gamma^2-1}{\gamma^2}$$

$$\gamma^2 = \frac{1}{1-\beta^2} = \frac{1+\beta^2}{(1-\beta^2)(1+\beta^2)} = \frac{1+\beta^2}{1-\beta^4}$$

We have:

$$\beta_{bp}^2 = \frac{\left(\beta_{tp}+\beta_{bt}\right)^2}{\left(1+\beta_{tp}\beta_{bt}\right)^2} = \frac{\beta_{tp}^2+2\beta_{tp}\beta_{bt}+\beta_{bt}^2}{1+2\beta_{tp}\beta_{bt}+\beta_{tp}^2\beta_{bt}^2}$$

$$\gamma_{bp}^2 = \frac{1}{1-\frac{\beta_{tp}^2+2\beta_{tp}\beta_{bt}+\beta_{bt}^2}{1+2\beta_{tp}\beta_{bt}+\beta_{tp}^2\beta_{bt}^2}} = \frac{1}{\frac{1+2\beta_{tp}\beta_{bt}+\beta_{tp}^2\beta_{bt}^2}{1+2\beta_{tp}\beta_{bt}+\beta_{tp}^2\beta_{bt}^2}+\frac{-\beta_{tp}^2-2\beta_{tp}\beta_{bt}-\beta_{bt}^2}{1+2\beta_{tp}\beta_{bt}+\beta_{tp}^2\beta_{bt}^2}}$$

$$= \frac{1+2\beta_{tp}\beta_{bt}+\beta_{tp}^2\beta_{bt}^2}{1+2\beta_{tp}\beta_{bt}+\beta_{tp}^2\beta_{bt}^2-\beta_{tp}^2-2\beta_{tp}\beta_{bt}-\beta_{bt}^2} = \frac{1+\beta_{tp}^2\beta_{bt}^2+2\beta_{tp}\beta_{bt}}{1+\beta_{tp}^2\beta_{bt}^2-\beta_{tp}^2-\beta_{bt}^2}$$ 🤮

Total energies:

in platform's frame: $E_p = \gamma_{tp}M_t c^2 + \gamma_{bp}M_b c^2$
in train's frame: $E_t = M_t c^2 + \gamma_{bt}M_b c^2$
in ball's frame: $E_b = \gamma_{bt}M_t c^2 + M_b c^2$

Quod demonstandum est: $E_p = E_t = E_b$

Step 1:

$$E_t = E_b \therefore M_t c^2 + \gamma_{bt}M_b c^2 = \gamma_{bt}M_t c^2 + M_b c^2$$

$$\gamma_{bt}M_b c^2 - M_b c^2 = \gamma_{bt}M_t c^2 - M_t c^2$$

$$(\gamma_{bt}-1)M_b c^2 = (\gamma_{bt}-1)M_t c^2$$

only possible if: $(M_b = M_t) \vee (\gamma_{bt} = 1 \therefore \beta_{bt} = 0)$

i.e. train & ball must have equal mass or the ball must be at rest within the train...

Ergo Conclusio: Total energy is frame-***de***pendent!

But (not proven *here*): $\boldsymbol{E^2 - p^2c^2 = m^2c^4}$ is frame-***in***dependent.