Einstein, A.
Zur Elektrodynamik bewegter Körper. (1905).
Annalen der Physik, 322(10), 891–921.

§ 5. Additionstheorem der Geschwindigkeiten.
(pp.905–907).

He first defines:

$$U^2 = \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2$$

and then comes up with:

$$U = \frac{\sqrt{(v^2+w^2+2vw\cos\alpha)-\left(\frac{vw\sin\alpha}{V}\right)^2}}{1+\frac{vw\cos\alpha}{V^2}}$$

Shame on you, Albert! This should of course be:

$$U = \pm\frac{\sqrt{v^2+w^2+2vw\cos\alpha-\left(\frac{vw\sin\alpha}{V}\right)^2}}{1+\frac{vw\cos\alpha}{V^2}}$$

or, if we radically eradicate the radical:

$$U^2 = \frac{v^2+w^2+2vw\cos\alpha-\left(\frac{vw\sin\alpha}{V}\right)^2}{\left(1+\frac{vw\cos\alpha}{V^2}\right)^2}$$

We derive:

$$\frac{U^2}{V^2} = \frac{\frac{v^2+w^2+2vw\cos\alpha}{V^2}-\frac{v^2w^2\sin^2\alpha}{V^2V^2}}{\left(1+\frac{v}{V}\frac{w}{V}\cos\alpha\right)^2}$$

We define:

$$\beta_i := \frac{v_i}{V}$$

yielding the **general equation** for *combining* velocities (it's not *addition*; no way!):

$$\boldsymbol{\beta_0^2 = \frac{\beta_1^2+\beta_2^2+2\beta_1\beta_2\cos\alpha-\beta_1^2\beta_2^2\,\sin^2\alpha}{(1+\beta_1\beta_2\cos\alpha)^2}}$$

For the *Lorentz factor* (which I'd rather call *time dilation factor*) we've got:

$$\boldsymbol{\gamma^2 = \frac{1}{1-\beta^2}}$$

hence:

$$\gamma^2-\gamma^2\beta^2 = 1 \therefore \gamma^2\beta^2 = \gamma^2 - 1 \therefore \boldsymbol{\beta^2 = \frac{\gamma^2-1}{\gamma^2}}$$

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If both ***velocities*** are ***parallel***, we have:

$$(\alpha = 0) \Rightarrow \cos\alpha = 1,\ \sin^2\alpha = 0$$

hence:

$$\beta_0^2 = \frac{\beta_1^2+\beta_2^2+2\beta_1\beta_2}{(1+\beta_1\beta_2)^2} = \frac{(\beta_1+\beta_2)^2}{(1+\beta_1\beta_2)^2} = \left(\frac{\beta_1+\beta_2}{1+\beta_1\beta_2}\right)^2$$

For the *physically correct* result we can simply remove both squares, yielding:

$$\boldsymbol{\beta_0 = \frac{\beta_1+\beta_2}{1+\beta_1\beta_2}}$$

i.e. ***sum by [one plus product]***.

Classical mechanics says: $\beta_0 = \beta_1 + \beta_2$

This combined velocity renders:

$$\boldsymbol{\gamma_0^2 = \frac{1}{1-\beta_0^2}} = \frac{1}{1-\frac{(\beta_1+\beta_2)^2}{(1+\beta_1\beta_2)^2}} = \frac{1}{\frac{(1+\beta_1\beta_2)^2-(\beta_1+\beta_2)^2}{(1+\beta_1\beta_2)^2}} = \frac{(1+\beta_1\beta_2)^2}{(1+\beta_1\beta_2)^2-(\beta_1+\beta_2)^2}$$

$$\boldsymbol{\gamma_0^2 = \frac{(1+\beta_1\beta_2)^2}{(1-\beta_1^2)(1-\beta_2^2)}} = \frac{1+\beta_1^2\beta_2^2+2\beta_1\beta_2}{1+\beta_1^2\beta_2^2-\beta_1^2-\beta_2^2}$$

Using:

$$\beta^2 = \frac{\gamma^2-1}{\gamma^2}$$

we obtain:

$$\boldsymbol{\gamma_0^2 = \gamma_1^2\gamma_2^2(1+\beta_1\beta_2)^2} = \gamma_1^2\gamma_2^2\left(1 \pm \frac{\sqrt{(\gamma_1^2-1)(\gamma_1^2-1)}}{\gamma_1\gamma_2}\right)^2$$

$$\boldsymbol{\gamma_0^2 = \left(\gamma_1\gamma_2 \pm \sqrt{(\gamma_1^2-1)(\gamma_1^2-1)}\right)^2}$$

where the ***sign*** should match that of the product $\beta_1\beta_2$.

Now we'll add some $\beta_2 = \beta$ to the very *speed of light* ($\beta_1 = 1$):

$$\beta_0 = \frac{1+\beta}{1+1\cdot\beta} = \frac{1+\beta}{1+\beta} = 1$$

so adding anything to the *speed of light* yields the *very same speed of light*!

Since addition to $\boldsymbol{c}$ yields $\boldsymbol{c}$, the *speed of light cannot* be *exceeded*!

Add anything? Well, $\beta = -1$ results in:

$$\beta_0 = \frac{1-1}{1-1} = \frac{\mathbf{0}}{\mathbf{0}}$$ 💀

IF a light source ***would*** be travelling ***at*** the very *speed of light* w.r.t. *any reference point* whatsoever, it would be *unable* to emit any light *backwards*, or, equivalently, a light source cannot emit towards an entity that's leaving it at the *speed of light*. Isn't this odd? Wouldn't this imply *achieving* the *speed of light* is *impossible*?

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For ***perpendicular velocities*** we get:

$$\left(\alpha = \pm\frac{\pi}{2}\right) \Rightarrow \cos\alpha = 0,\ \sin^2\alpha = 1$$

and then:

$$\boldsymbol{\beta_0^2 = \beta_1^2 + \beta_2^2 - \beta_1^2\beta_2^2}$$

Classical mechanics says: $\beta_0^2 = \beta_1^2 + \beta_2^2$

Since we're considering perpendicular motions,
any sign seemingly seems to seem seemingly senseless.

We also find:

$$\boldsymbol{\beta_0^2} = \frac{\boldsymbol{\gamma_0^2 - 1}}{\boldsymbol{\gamma_0^2}} = \frac{\gamma_1^2-1}{\gamma_1^2} + \frac{\gamma_2^2-1}{\gamma_2^2} - \frac{(\gamma_1^2-1)(\gamma_2^2-1)}{\gamma_1^2\gamma_2^2}$$

$$= \frac{(\gamma_1^2-1)\gamma_2^2}{\gamma_1^2\gamma_2^2} + \frac{\gamma_1^2(\gamma_2^2-1)}{\gamma_1^2\gamma_2^2} - \frac{\gamma_1^2\gamma_2^2-\gamma_1^2-\gamma_2^2+1}{\gamma_1^2\gamma_2^2}$$

$$= \frac{\gamma_1^2\gamma_2^2-\gamma_2^2}{\gamma_1^2\gamma_2^2} + \frac{\gamma_1^2\gamma_2^2-\gamma_1^2}{\gamma_1^2\gamma_2^2} + \frac{-\gamma_1^2\gamma_2^2+\gamma_1^2+\gamma_2^2-1}{\gamma_1^2\gamma_2^2}$$

$$= \frac{\gamma_1^2\gamma_2^2-\gamma_2^2+\gamma_1^2\gamma_2^2-\gamma_1^2-\gamma_1^2\gamma_2^2+\gamma_1^2+\gamma_2^2-1}{\gamma_1^2\gamma_2^2}$$

leaving us with:

$$\beta_0^2 = \frac{\gamma_0^2-1}{\gamma_0^2} = \frac{\gamma_1^2\gamma_2^2-1}{\gamma_1^2\gamma_2^2} \therefore \boldsymbol{\gamma_0^2 = \gamma_1^2\gamma_2^2}$$

so, for ***perpendicular velocity combination***, we have:

$$\boldsymbol{\gamma_0 = \gamma_1\gamma_2}$$

i.e. ***time dilation factors multiply***.

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Cosmic truths:
https://henk-reints.nl/u/index.php

Gravitation without the use of tensors:
https://henk-reints.nl/u/notensors.php

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