Define:

$$z \coloneqq \cos\varphi + i\sin\varphi$$

$$\varphi = 0 \Rightarrow z = \cos 0 + i\sin 0 = 1 + 0i = 1$$

$$\frac{dz}{d\varphi} = -\sin\varphi + i\cos\varphi$$

$$\therefore \frac{dz}{d\varphi} = i^2\sin\varphi + i\cos\varphi$$

$$\therefore \frac{dz}{d\varphi} = i(i\sin\varphi + \cos\varphi)$$

$$\therefore \frac{dz}{d\varphi} = i(\cos\varphi + i\sin\varphi)$$

$$\therefore \frac{dz}{d\varphi} = iz$$

$$\therefore \frac{dz}{z} = i d\varphi$$

$$\therefore \int\frac{dz}{z} = i\int d\varphi + C$$

$$\therefore \ln z = i\varphi + C$$

$$\therefore e^{\ln z} = e^{i\varphi + C}$$

$$\therefore z = e^{i\varphi}e^{C}$$

$$\therefore \varphi = 0 \Rightarrow z = e^{C}$$

also: $\varphi = 0 \Rightarrow z = 1$

$$\therefore e^{C} = 1$$

$$\therefore z = e^{i\varphi}$$

$$e^{i\varphi} = \cos\varphi + i\sin\varphi$$

Quod Erat Demonstrandum.

The above starts with the outcome that
Euler himself presumably did not yet know.
He actually found it (back in 1748) through the series
expansions of exponential and trigonometric functions.