Newsteinian *gravitational potential energy*[1] in a homogeneous sphere:

$$\boldsymbol{E_{\text{pot,NS,sphere}}} = \frac{-3GM^2}{5R_{\text{prop}}}$$

which in ***proper space*** simply is *Newtonian*.

The document in footnote 1 *gives a detailed description of this terminology and what it means.*

Below, ***density*** & ***radii*** are in ***proper space*** unless stated otherwise.

Suppose two equal spheres merge into a new sphere
with the same ***proper density*** $\boldsymbol{\Omega}$ as both originals.

Newton, *very beginning of the* Principia, pag. I., Definitio I.:
mass or ***body*** = ***amount of matter*** (quantitas materiæ).

Since the total no. of nucleons will not change, the spheres' ***masses*** simply add up: $M_1 = 2M_0$

As the ***proper density*** persists, the ***proper volumes*** add up too: $\frac{4\pi}{3}R_1^3 = 2\frac{4\pi}{3}R_0^3 \therefore R_1 = \sqrt[3]{2}R_0$

Total $\boldsymbol{E_{\text{pot}}}$ ***before*** the merger: $E_{\text{pot},0} = 2\frac{-3GM_0^2}{5R_0} = \frac{-6GM_0^2}{5R_0}$

Total $\boldsymbol{E_{\text{pot}}}$ ***after*** the merger: $E_{\text{pot},1} = \frac{-3GM_1^2}{5R_1} = \frac{-3G(2M_0)^2}{5\sqrt[3]{2}R_0} = \frac{-12GM_0^2}{5\sqrt[3]{2}R_0}$

Their difference: $\Delta E_{\text{pot}} = E_{\text{pot},1} - E_{\text{pot},0} = \frac{-12GM_0^2}{5\sqrt[3]{2}R_0} - \frac{-6GM_0^2}{5R_0}$

equals: $\Delta E_{\text{pot}} = \frac{6(\sqrt[3]{2}-2)GM_0^2}{5\sqrt[3]{2}R_0} = \frac{-6(2-\sqrt[3]{2})GM_0^2}{5\sqrt[3]{2}R_0} = \frac{2-\sqrt[3]{2}}{\sqrt[3]{2}} \cdot \frac{-6GM_0^2}{5R_0}$

We calculate: $\frac{2-\sqrt[3]{2}}{\sqrt[3]{2}} = \frac{\sqrt[3]{8}}{\sqrt[3]{2}} - \frac{\sqrt[3]{2}}{\sqrt[3]{2}} = \sqrt[3]{4} - 1 \approx 0.587\,401\,051\,968\,199\,5$

and obtain: $\boldsymbol{\Delta E_{\text{pot,merger}} = \left(\sqrt[3]{4}-1\right)\frac{-6GM_0^2}{5R_0} = \left(\sqrt[3]{4}-1\right)E_{\text{pot},0}}$

Since this is negative, it is an energy *release*. Cf. a stone that is falling on your head, releasing potential energy that will ***HURT YOU*** 👊👿.

But didn't this stone actually release pot. energy that it had relative to your head?
Didn't we until now consider only the merger itself?
We have not yet taken into account the potential energy "between" the ***masses***, which will of course also be released.
So let's calculate the pot. energy between two touching equal spheres.

Eqn.[16] (as of 2026-04-15) of doc. in footnote 1: $E_{\text{pot}} = \frac{c^2 Mm \ln\frac{rc^2-GM}{rc^2-Gm}}{M-m}$ where $\boldsymbol{r}$ is the intermediate distance in ***expanded space***.

It also gives: $\lim\limits_{m\to M} E_{\text{pot}} = \frac{GM^2c^2}{GM-rc^2}$

For equal ***masses***, both ***proper spaces*** have the same ***grav. expansion factor***.
Since both $\boldsymbol{M}$s are equal, we can convert to ***proper space*** using the same $\boldsymbol{GM/c^2}$, i.e.: $\boldsymbol{r = r_{\text{prop}} + \frac{GM}{c^2}}$

[1] https://henk-reints.nl/astro/HR-Newstein-corrected.pdf

hence: $$E_{\text{pot}} = \frac{GM^2c^2}{GM - rc^2} = \frac{GM^2c^2}{GM - \left(r_{\text{prop}} + \frac{GM}{c^2}\right)c^2} = \frac{GM^2c^2}{GM - r_{\text{prop}}c^2 - GM} = \frac{-GM^2}{r_{\text{prop}}}$$

Doesn't ref.1 mention: in ***proper space***, we have ***Newtonian gravitation*****?**

We end up with: $$E_{\text{pot,intermediate}} = \frac{-GM_0^2}{2R_0}$$

This (negative) energy should of course be added to the pre-merger energy, hence subtracted from the difference found sofar, yielding

a total energy release of: $$E_{\text{released}} = -\Delta E_{\text{pot,merger}} - E_{\text{pot,intermediate}}$$

$$= \frac{6\left(\sqrt[3]{4}-1\right)GM_0^2}{5R_0} + \frac{GM_0^2}{2R_0}$$

Therefore: $$E_{\text{released}} = \frac{12\sqrt[3]{4}-7}{10}\frac{GM_0^2}{R_0} \approx 1.205\frac{GM_0^2}{R_0}$$

One may presume this energy is released as ***some*** combination of ***electromagnetic radiation***, ***gravitational waves*** & ***internal thermal energy***.

Note: I claim ***gravitational waves*** are ***longitudinal*** Quilída[2] ***tenuity waves*** and ***not*** something inscrutable about the **non existing** fabric of spacetime which is a *fully abstract* concept that lives in Homo Excogitans' mind only.

Sapiens? Ho maar![3]

We also derive:

$$E_{\text{rel}} = \frac{12\sqrt[3]{4}-7}{10}\frac{GM_0^2}{R_0} = \frac{12\sqrt[3]{4}-7}{10}\frac{5}{3}\frac{3GM_0^2}{5R_0} = \frac{12\sqrt[3]{4}-7}{6}\frac{3GM_0^2}{5R_0} = \frac{12\sqrt[3]{4}-7}{6}\left|E_{\text{pot,single}}\right|$$

$$\frac{12\sqrt[3]{4}-7}{6} \approx 2.008\,135\,437$$

$$M_0 = \frac{M_1}{2},\quad R_0 = \frac{R_1}{\sqrt[3]{2}} \quad \therefore \frac{GM_0^2}{R_0} = \frac{\sqrt[3]{2}}{4}\frac{GM_1^2}{R_1}$$

$$E_{\text{rel}} = \frac{12\sqrt[3]{4}-7}{10}\frac{\sqrt[3]{2}}{4}\frac{GM_1^2}{R_1} = \frac{24-7\sqrt[3]{2}}{40}\frac{GM_1^2}{R_1} = \frac{120-35\sqrt[3]{2}}{72}\frac{3GM_1^2}{5R_1} = \frac{120-35\sqrt[3]{2}}{72}\left|E_{\text{pot,merged}}\right|$$

$$\frac{120-35\sqrt[3]{2}}{72} \approx 1.054\,205\,045$$

Apparent final ***mass*** loss as a fraction:

$$\frac{E_{\text{rel}}}{M_1c^2} = \frac{24-7\sqrt[3]{2}}{40}\frac{G}{c^2}\frac{M_1}{R_1} = \frac{24-7\sqrt[3]{2}}{40}\frac{G}{c^2}\frac{M_1}{\sqrt[3]{\frac{3M_1}{4\pi\Omega}}} = \frac{24-7\sqrt[3]{2}}{40}\frac{G}{c^2}\sqrt[3]{\frac{4\pi\Omega M_1^2}{3}}$$

where $\boldsymbol{\Omega}$ is the ***proper density*** (which was presumed unchanged by the merger).

Close-packed neutronium:

$\Omega_{n,\text{cp}} \approx 5.0 \times 10^{17}$ kg/m³ (see derivation further below)

$$\frac{E_{\text{rel}}}{M_{\text{final}}c^2} = \frac{24-7\sqrt[3]{2}}{40}\cdot\sqrt[3]{\frac{4\pi G^3\Omega_{n,\text{cp}}M_\odot^2}{3c^6}\frac{M_{\text{final}}^2}{M_\odot^2}} = \frac{24-7\sqrt[3]{2}}{40}\cdot\sqrt[3]{\frac{4\pi G^3\Omega_{n,\text{cp}}M_\odot^2}{3c^6}}\cdot\sqrt[3]{N_{\odot,\text{final}}^2}$$

$$\approx 0.0570\cdot\sqrt[3]{N_{\odot,\text{final}}^2}$$

[2] https://henk-reints.nl/astro/HR-Deflection-of-light-passing-a-mass.pdf
[3] That's a Dutch pun. "*Ho maar*" here means something like: "*I wouldn't think so*", hence: *Wise? I think not!*

Example: two merging neutron stars of solar mass having the density of close-packed tiny little marbles.

Close-packing factor: $\frac{V_{\text{gross}}}{V_{\text{net}}} = \frac{3\sqrt{2}}{\pi}$

We have[4]: $\mathbf{r_n \lesssim 0.84 \times 10^{-15}\ m}$ (two significant digits)

yielding: $V_{n,\text{net}} \approx 2.483 \times 10^{-45}\ \text{m}^3$ (a bit more precise to

hence: $V_{n,\text{gross}} \approx 3.353 \times 10^{-45}\ \text{m}^3$ avoid rounding errors)

With: $m_n \approx 1.674\,927\,500\,56 \times 10^{-27}\ \text{kg}$ (CODATA 2022)

we obtain: $\mathbf{\Omega_{n,cp} \approx 5.0 \times 10^{17}\ kg/m^3}$ (two significant digits)

rendering: $R = \sqrt[3]{3M_{\odot}/4\pi\Omega_{n,cp}}$

yielding: $R \approx 9.83\ \text{km}$

which we round to: $\mathbf{R = 10^4\ m}$

Then: $E_{\text{released}} = \frac{12\sqrt[3]{4}-7}{10}\frac{GM_{\odot}^2}{10\ \text{km}} \approx \mathbf{3.18 \times 10^{46}\ J}$

Equivalent *mass*: $\sim\mathbf{3.54 \times 10^{29}\ kg} \approx \mathbf{0.18 M_{\odot}}$

Since we ended up with $2M_{\odot}$, we ***apparently lost*** $\sim 9\%$ during the merger.

Obviously, the found: $\frac{E_{\text{rel}}}{M_{\text{final}}c^2} \approx 0.0570 \cdot \sqrt[3]{N_{\odot,\text{final}}^2} \approx 0.0570 \cdot \sqrt[3]{2^2}$ also yields $\sim\mathbf{9.0\%}$.

Of course any ***mass measurement*** before the merger already ***included*** this ***equivalent mass*** (but the above derivations used bare masses, i.e. without it; inclusion would significantly complicate the derivation). Hence, a huge ***mass seemingly*** disappears during the merger. As a matter of fact, ***zero mass*** is actually converted to ***energy***, since ***no antimatter*** is involved at all.

Moreover, this ***equivalent mass*** actually is ***inertia***, which is a ***property*** of ***mass***
(Newton, Principia, Definitio III. literally mentions "inertia massæ")
& Einstein found ***inertia*** is a ***property*** of ***energy*** as well.
$\boldsymbol{E = mc^2}$ does not say ***energy is mass***, but ***energy*** ***<u>has</u>*** ***inertia***
and, similarly, *relativistic* ***mass*** actually is *relativistic* ***inertia***.

Mass is ***amount of matter*** ≈ ***no. of nucleons*** (for which I coined a better name: *corpions*[5]),
(Newton, Principia, Definitio I.: Quantitas materiæ est mensura etc.)
which evidently is ***independent*** of any ***velocity***!

Mass *is not a* ***property*** *but an* ***entity***. **Inertia massæ** *is a* ***property*** *thereof.*

Thou shalt not cosnufe Quantitas Materiæ (mass) and Inertia Massæ!

Now consider *the* ***Higgs field***. *Does it really give* ***particles*** *their* ***mass****? Isn't* ***mass*** *an amount of* ***matter****?*
Isn't ***matter*** *a collection of* ***particles****? Isn't* ***mass*** *then a no. of* ***particles****? Isn't a* ***particle*** *a (tiny little)* ***body****?*
Read **Definitio I.** *in the* **Principia** *and see that the terms* ***body*** *&* ***mass*** *are defined* ***fully*** *synonymously*[6]***!***
Particles ***have*** *no* ***mass****, they* ***ARE mass****!* ***Higgs*** *gives* ***mass*** *its* ***inertia!*** ***Higgs*** *gives* ***matter*** *its* ***inertia!***

thou shalt not say mass when thou meanst inertia! (& the latter is practically *always* true!)
& you should ***NOT TAMPER*** *with Newton's or whomever's DEFINITIONS !*
If you mean something else, then coin another name!
Per Iovem & Toutatem & Isim & Osirim & totum gregem!
By Jupiter & Toutatis & Isis & Osiris & the whole flock!

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[4] Actually, the neutron mass is known with a precision of merely a single digit,
but in https://henk-reints.nl/HR-Existance-postulate.pdf I derive a more precise value.
However, I *cannot* and *will not* claim it as an *ascertained truth*, although I consider it very plausible.

[5] A *corpion* would essentially be a proton+electron pair or a neutron, i.e. that what makes up persistent matter.

[6] See https://henk-reints.nl/astro/HR-acceleration-gravitation-geodesic.pdf p.5 (as of 2026-04-14).

