In https://henk-reints.nl/astro/HR-Newstein-corrected.pdf, Addendum 2026-07-23, I found:

Total dimensionless specific energy (i.e. *Lorentz factor*)

of a body circularly orbiting at $\boldsymbol{\rho_M = \frac{rc^2}{GM}}$ equals: $\boldsymbol{\gamma_{tot}^2 = \frac{\rho_M^4 - \rho_M^3 + \rho_M^2}{(\rho_M - 1)^4}}$

In https://henk-reints.nl/astro/HR-Newsteinian-energy-escaping-light.pdf, I easily derived:

Light has: $\boldsymbol{\gamma_\gamma = 2}$

then: $\rho_M^4 - \rho_M^3 + \rho_M^2 = 2^2(\rho_M - 1)^4 = 4(\rho_M^4 - 4\rho_M^3 + 6\rho_M^2 - 4\rho_M + 1)$

i.e.: $\rho_M^4 - \rho_M^3 + \rho_M^2 = 4\rho_M^4 - 16\rho_M^3 + 24\rho_M^2 - 16\rho_M + 4$

hence: $4\rho_M^4 - 16\rho_M^3 + 24\rho_M^2 - 16\rho_M + 4 - \rho_M^4 + \rho_M^3 - \rho_M^2 = 0$

implying: $\boldsymbol{3\rho_M^4 - 15\rho_M^3 + 23\rho_M^2 - 16\rho_M + 4 = 0}$

This is a ***quartic equation***: $\boldsymbol{ax^4 + bx^3 + cx^2 + dx + e = 0}$

with: $\boldsymbol{a = 3,\ b = -15,\ c = 23,\ d = -16,\ e = 4}$

One of its ***root***s should be the ***radius*** where ***light*** would ***orbit***, while the others ***probably*** are ***physically meaningless*** (either *complex* or $\rho_M < 1$ (or yielding $\gamma \neq 2$)).

WolframAlpha (as well as Google *AI*) both yield:

$\rho_{M;0} = 0.523\,705\,770\,617\,694\,85 < 1$

$\boldsymbol{\rho_{M;1} = 2.974\,846\,281\,364\,383\,8}$

$\rho_{M;2,3} = 0.750\,723\,974\,008\,960\,70 \pm 0.540\,594\,319\,042\,811\,85\,i$

Hence, light would circularly orbit

at a radius of: $\boldsymbol{\sim 2.974\,846\,r_M}$

being: $\boldsymbol{\sim 1.487\,423\,r_S}$

Smaller orbits should definitely be possible,
but they require a larger *Lorentz factor* that what light has.

The *Lorentz factor* of *light* equals *TWO*!

Not more, not less!

See https://henk-reints.nl/astro/HR-Newsteinian-energy-escaping-light.pdf.

Found radius is less than the *photon sphere* according to the *convential black hole equation*, which is an *incorrect* form of the Schwarzschild solution. When we correct that (see https://henk-reints.nl/astro/HR-True-Schwarzschild-Solution.pdf) we obtain:

$$r = \sqrt[3]{(1.5)^3 - 1} \approx 1.334\,200\,824\,360\,972\,4$$

which neither is the correct value. As explained in said link,
Karl Schwarzschild has found a *correct solution* of the ***flawed*** *Einstein equation*,
please read https://henk-reints.nl/astro/HR-acceleration-gravitation-geodesic.pdf.
The corresponding *photon sphere* is then inherently wrong as well.
I claim it actually equals the above value.

As far as we know, our computer has never had an undetected error.
— Conrad Weisert —

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FROM THE MAKERS OF WOLFRAM LANGUAGE AND MATHEMATICA

WolframAlpha

roots of 3x^4-15x^3+23x^2-16x+4=0

Natural Language | Math Input

Input interpretation:

roots $3x^4 - 15x^3 + 23x^2 - 16x + 4 = 0$

Results: Fewer digits | More digits | Exact forms | Step-by-step solution

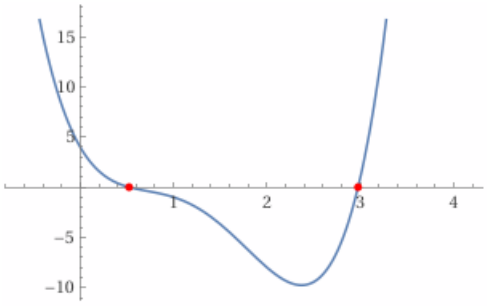
$x \approx 0.52370577061769484630000154$

$x \approx 2.9748462813643838142353 65$

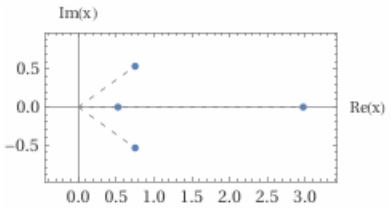
$x \approx 0.75072397400896066973230 98 - 0.54059431904281184588771 94\,i$

$x \approx 0.75072397400896066973230 98 + 0.54059431904281184588771 94\,i$

Root plot:



Roots in the complex plane:



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