N.B.: *In this very document, I am only calculating some possibilities, without* claiming *anything about the true nature of an electron. I'm merely pondering the particle as simply as possible.*

Common sense is not so very common.

— attributed to Voltaire —

In Dutch, *common sense* is called: *gezond verstand* = *healthy sense*, yielding *my* interpretation:

Gezond verstand is vrij vaak ziek.

Healthy sense is quite often ill.

Henk Reints:

Assumptions always sprout from nescience,
hence, they have nothing to do with science.

Latin *scientia* = *know*ledge.

I do not claim this document is high-level state-of-the-art science.

What is electric charge?

To me, it merely is a completely *un*understood property that causes bodies to behave in agreement with Coulomb's law. Full stop.

Well, we have found out that on the macroscopic level, it is a surplus or shortage of electrons. The latter actually is a proton surplus.
In this situation, it can be considered a fluid with some density distribution etc.
But isn't the electron itself an ***atomic charge***, i.e. indivisible?
Based on which ascertained truth can we say *anything* about its internal charge distribution?
Well, it ***does*** have a measurable magnetic moment,
so it ***does*** behave like ***some*** circular current.
Therefore, it *seems* that its charge distribution *cannot* be point-like,
dspite all theories that are denying a truly spinning larger than zero charge.

PRESUMPTION (picked from thin air!):

An electron is a homogeneous solid sphere
of uniform mass & charge density.

This simply is the premise I'm going to use below
& I'll mostly ignore the effects of moving or accelerating charge.
I'm merely 🎻🎼🎶♪♫♬ around, pondering, speculating & expressing ideas that may be unable to leave your mind once you've taken notice of them 😎.

By the way: Demonstration of Spin 1/2
gives a nice visualisation of how the elctron spin would work,
but please keep in mind: omnis comparatio claudicat, every comparison is lame.

Let's go!

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Moment of inertia of a sphere: $I = kMR^2$

for a homogeneous solid sphere: $k = \frac{2}{5}$

Spin angular momentum: $L = I\omega = kMR^2\omega$

relativistic (homog. solid sph.)[1]: $\boldsymbol{L_{\mathrm{rel}}} = \frac{kMR^2\omega}{\left(1-k\left(\frac{\omega R}{c}\right)^2\right)^{3/2}} = \boldsymbol{\frac{kMRc\beta_R}{\left(1-k\beta_R^2\right)^{3/2}}}$

where: $\beta_R = \frac{\omega R}{c}$ ("equator velocity")

Two lines of quantum mechanics:

Electron spin: $s = \frac{1}{2}$

Corresp. angular momentum: $S = \sqrt{s(s+1)}\hbar$ (*presuming* they got this right...)

Henk **h**eavily **h**ates $\hbar$, therefore: $\boldsymbol{S} = \sqrt{s(s+1)}\,\frac{h}{2\pi} = \frac{1}{2}\sqrt{3}\,\frac{h}{2\pi} = \boldsymbol{\frac{\sqrt{3}h}{4\pi}}$

When $\boldsymbol{4\pi}$ appears, I always immediately think of something spherical!

The very same applies to Heisenberg's uncertainty principle if one replaces $\hbar/2$ with $h/4\pi$! Doesn't this look like an *isotropic angular momentum per solid angle*? & I often see: $\hbar$ m'plied by some multiple of π. Which *lucid mind* invented that?

Apparently: $S = \sqrt{3} \times$ Heisenberg uncertainty. I know $\sqrt{3}$ from geometry. & I consider a full rotation[2] way more fundamental than a radian, so no $/2\pi$, please.

On https://www.physicsforums.com/threads/calculate-the-magnetic-moment-of-a-rotating-sphere.1059787/ (last two entries by LeoJakob & TSny of Feb 7, 2024),

I find for the magnetic moment of a spinning homog. solid sphere: $\vec{\mathcal{M}} = \frac{1}{5}QR^2\vec{\omega}$ (cf. "spinnergy" eqn. on p.7)

as well as: $\vec{\mathcal{M}} = \frac{Q}{2M}\vec{L}$

For the electron, the latter yields: $\boldsymbol{\mathcal{M}_e = \frac{q_e}{2m_e}\frac{\sqrt{3}h}{4\pi} \approx -8.03152831 \times 10^{-24}\ \mathrm{J/T}}$

However, CODATA 2022 says: $\boldsymbol{\mathcal{M}_e \approx -9.2847646917 \times 10^{-24}\ \mathrm{J/T}}$

which is: 1.15603959 times larger.

Could this indicate a somewhat flattened sphere (like the earth) or so?

We now equate: $S = L_{\mathrm{rel}}$

hence: $\frac{\sqrt{3}h}{4\pi} = \frac{km_eRc\beta_R}{\left(1-k\beta_R^2\right)^{3/2}}$

yielding: $R = \frac{\sqrt{3}\left(1-k\beta_R^2\right)^{3/2}h}{4\pi km_ec\beta_R} = \frac{\sqrt{3}\left(1-k\beta_R^2\right)^{3/2}\lambda_{\mathrm{C},e}}{4\pi k\beta_R}$

therefore: $\boldsymbol{\frac{R}{\lambda_{\mathrm{C},e}} = \frac{\sqrt{3}\left(1-k\beta_R^2\right)^{3/2}}{4\pi k\beta_R}}$

With said $k = \frac{2}{5}$, this becomes: $\boldsymbol{R_* := \frac{R}{\lambda_{\mathrm{C},e}} = \frac{\sqrt{3}\left(1-\frac{2}{5}\beta_R^2\right)^{3/2}}{4\pi\frac{2}{5}\beta_R} = \frac{\sqrt{3}\left(5-2\beta_R^2\right)^{3/2}}{8\sqrt{5}\pi\beta_R}}$

WolframAlpha gives several exact solutions for its inverse: $\beta_R(R_*)$, but why does it come up with such a complicated mess that can still be significantly simplified? The only (further simplified by myself) positive real solution it gives, is:

[1] See e.g.: https://www.sjsu.edu/faculty/watkins/relangularmom.htm search: "The relativistic angular momentum L is".

[2] See: https://henk-reints.nl/HR-circular-motion.pdf

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$$\beta_R = \sqrt{\frac{5}{2}+\frac{1}{3}\left(\sqrt[3]{10\pi^2R_*^2\left(\sqrt{5(128\pi^2R_*^2+405)}-45\right)}-\frac{40\pi^2R_*^2}{\sqrt[3]{10\pi^2R_*^2\left(\sqrt{5(128\pi^2R_*^2+405)}-45\right)}}\right)}$$

From the just-found eqn. for R_*, we easily find: $\beta_R = 1 \Rightarrow \boldsymbol{R_* = \frac{9}{8\sqrt{5}\pi} \approx 0.16015}$

which, due to the *speed limit of light*, would be the minimal possible radius of a spinning solid & homogeneous sphere with mass & spin angular momentum equal to those of an electron, expressed in units of its Compton wavelength.

We easily find: $R_{*,\min} = \frac{9}{8\sqrt{5}\pi} \Rightarrow R_{e,\min} \approx 689\ \text{fm},$

whereas the *classical electron radius* is: 2.817 940 3205 fm.

We also have: $R_* = \frac{1}{2} \Rightarrow \beta_R = \sqrt{\frac{5}{2}+\frac{1}{3}\left(\sqrt[3]{\frac{5}{2}\pi^2\left(\sqrt{5(32\pi^2+405)}-45\right)}-\frac{10\pi^2}{\sqrt[3]{\frac{5}{2}\pi^2\left(\sqrt{5(32\pi^2+405)}-45\right)}}\right)}$

which equals: $\boldsymbol{\beta_R \approx 0.56254}$

and this can of course also be found numerically from the original equation. It is the equator velocity if the electron *would* homogeneously occupy its entire *Compton sphere*[3], which has its *diameter* equal to the Compton wavelength. This now found *Compton equator velocity* does not contradict any law of physics that I know of.

In spite of the found: $R_{*,\min} = \frac{9}{8\sqrt{5}\pi}$

my Existance Postulate[3] implies the *diameter* $= 2R$ should exceed the Compton wavelength. Moreover, stable (non-decaying) particles ***might*** necessarily be smaller than[3]: $\frac{4}{\pi}\lambda_C$, hence: $\boldsymbol{1 \le \left(\frac{2R}{\lambda_C} = 2R_*\right) < \frac{4}{\pi}}$

For an electron, this would mean:

$$2.426\ 310\ 235\ 38\ \text{pm} < 2R < 3.089\ 274\ 139\ 48\ \text{pm}$$

& this upper boundary yields: $\boldsymbol{\beta_R \approx 0.47088}$

which, like the said $\beta_R \approx 0.56254$, seems totally plausible, seemsn't it?

WHAT IF it would actually spin such that exactly: $\boldsymbol{\beta_R \equiv 1/2}$ **?** ⇐⇐ **SPECULATION!**

Wouldn't its *diameter* $2R_* = 2\frac{\sqrt{3}\left(5-2\beta_R^2\right)^{3/2}}{8\sqrt{5}\pi\beta_R} = \frac{27\sqrt{3}}{4\sqrt{2}\sqrt{5}\pi} \approx 1.17683$

then nicely sit between said limits of $\boldsymbol{1}$ & $\boldsymbol{4/\pi} \approx 1.27324$,

turning it into an existing[3] stable[3] particle**?**

[3] See: https://henk-reints.nl/HR-Existance-postulate.pdf

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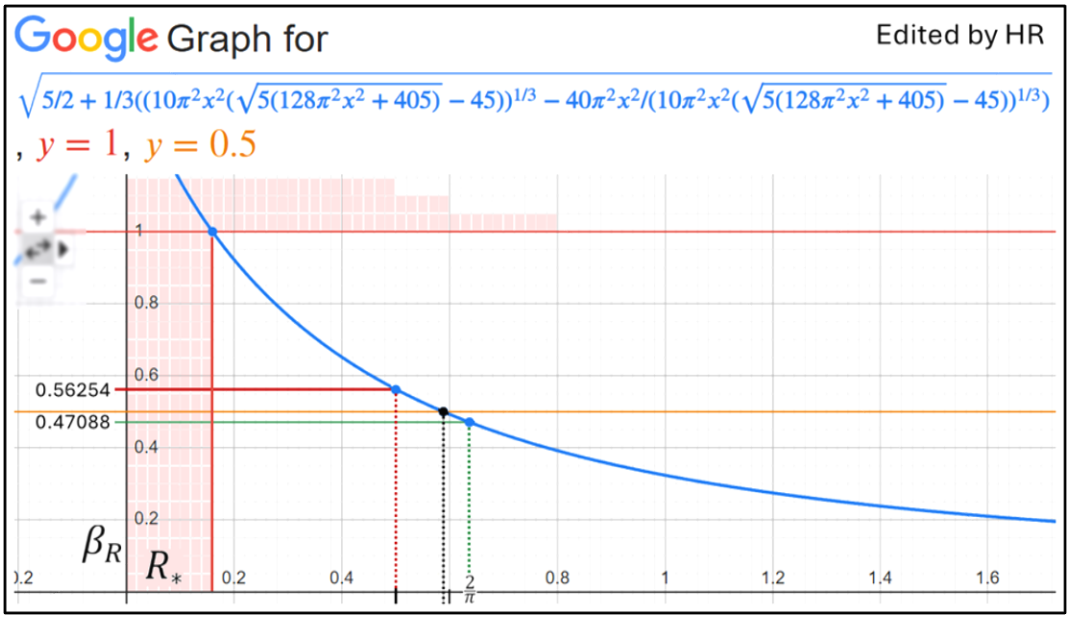

For the centripetal acceleration,

we find: $$a_{\text{cp,spin}} = \frac{v^2}{r} = \frac{c^2\beta_R^2}{\lambda_{\text{C},e}R_*} = \frac{m_e c^3}{h}\frac{\beta_R^2}{R_*} = \frac{m_e c^3}{h}\frac{\beta_R^2}{\frac{\sqrt{3}(5-2\beta_R^2)^{3/2}}{8\sqrt{5}\pi\beta_R}} = \frac{m_e c^3}{h}\frac{8\sqrt{5}\pi\beta_R^3}{\sqrt{3}(5-2\beta_R^2)^{3/2}}$$

We *choose*: $\beta_R = \frac{1}{2}$

& obtain: $$a_{\text{cp,spin}} = \frac{m_e c^3}{h}\frac{8\sqrt{5}\pi\cdot\frac{1}{8}}{\sqrt{3}\left(5-2\cdot\frac{1}{4}\right)^{3/2}} = \frac{m_e c^3}{h}\frac{2\sqrt{2}\sqrt{5}\pi}{27\sqrt{3}} \approx 8.879\times10^{27}\ (\text{m/s}^2 = \text{N/kg})$$

The (dimensionless) velocity of an electron orbiting a proton at the Bohr radius equals the fine structure constant,

yielding: $$a_{\text{cp,orb}} = \frac{c^2\alpha^2}{a_0} \approx 9.044\times10^{22}\ (\text{m/s}^2 = \text{N/kg})$$

hence: $$\frac{a_{\text{cp,spin}}}{a_{\text{cp,orb}}} \approx 98\,400 \approx 100\,000$$

The centripetal force on an orbiting electron

would be: $$F_{\text{cp,orb}} = \frac{c^2\alpha^2}{a_0}m_e \approx 8.238\,7235\times10^{-8}\ \text{N}$$

Let this force be exerted on the projected surface

of a sphere with: $$2R_* = \frac{4}{\pi} \therefore R = \frac{2h}{\pi m_e c}$$

hence: $$A = \pi\left(\frac{2h}{\pi m_e c}\right)^2 = \frac{4h^2}{\pi m_e^2 c^2}$$ **(in this calc., e = elem. charge)**

yielding: $$p_{\text{cp,orb}} = \frac{\frac{c^2\alpha^2}{a_0}m_e}{\frac{4h^2}{\pi m_e^2 c^2}} = \frac{\pi\alpha^2 m_e^3 c^4}{4a_0 h^2} = \frac{\pi\left(\frac{e^2}{2\varepsilon_0 hc}\right)^2 m_e^3 c^4}{4\left(\frac{4\pi\varepsilon_0\hbar^2}{e^2 m_e}\right)h^2} = \frac{\pi\frac{e^4}{4\varepsilon_0^2 h^2 c^2}m_e^3 c^4}{16\pi\varepsilon_0\frac{h^2}{4\pi^2}\frac{h^2}{e^2 m_e}}$$

$$= \frac{\pi e^6 m_e^4 c^4}{4\varepsilon_0^2 h^2 c^2\left(16\pi\varepsilon_0\frac{h^2}{4\pi^2}\right)h^2} = \frac{\pi^2 e^6 m_e^4 c^2}{16\varepsilon_0^3 h^6} = \frac{\pi^2\mu_0^3 e^6 m_e^4 c^8}{16h^6}$$

$$= \left(\frac{\pi}{4}\right)^2\left(\frac{\mu_0 e^2}{h^2}\right)^3 (m_e c^2)^4$$

$\approx 1.099\times10^{16}$ Pa $\leq p_{\text{C},e}$ = electron Compton pressure[3],

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which means the electron will *not* get crushed by the orbital centri{*pet*/*fug*}al forces, which nicely agrees with the known fact that the hydrogen atom is stable.

We will now derive the relativistic (rotational) kinetic energy ("kinergy") of a spinning homogeneous solid sphere. See further below @p.10 (where I do calculations for a homogeneous hollow sphere, which I wrote earlier than *this* very section) for details about integrating over a spherical surface and the symbols I chose for the angular coordinates (well, θ = colatitude & φ = longitude).

The velocity of any infinitesimal part of a spherical shell equals: $v = \omega r \sin\theta$

and, going dimensionless: $\beta = \beta_r \sin^2\theta$ where: $\beta_r = \frac{\omega r}{c}$

Its infinitesimal mass is: $d^3M = \rho r^2 \sin\theta \, d\varphi d\theta dr$

where: ρ = the density of the solid sphere

Its relativistic "kinergy" is: $d^3E_k = (\gamma - 1)c^2 d^3M = c^2\left(\frac{1}{\sqrt{1-\beta^2}} - 1\right) d^3M$

i.e.: $d^3E_k = c^2\left(\frac{1}{\sqrt{1-\beta_r^2\sin^2\theta}} - 1\right) d^3M$

therefore: $d^3E_k = \rho c^2 r^2 \frac{1-\sqrt{1-\beta_r^2\sin^2\theta}}{\sqrt{1-\beta_r^2\sin^2\theta}} \sin\theta \, d\varphi d\theta dr$

from which we obtain: $E_k = \rho c^2 \int_0^R r^2 \int_0^\pi \frac{1-\sqrt{1-\beta_r^2\sin^2\theta}}{\sqrt{1-\beta_r^2\sin^2\theta}} \sin\theta \int_0^{2\pi} d\varphi \, d\theta \, dr$

which easily becomes: $E_k = 2\pi\rho c^2 \int_0^R r^2 \int_0^\pi \frac{1-\sqrt{1-\beta_r^2\sin^2\theta}}{\sqrt{1-\beta_r^2\sin^2\theta}} \sin\theta \, d\theta \, dr$

WolframAlpha ***does*** come up with the ***in***definite red integral, *but it seems incapable of finding the* ***definite*** *integral by simply subtracting both boundary values* 😳. 🙌 done, Mr. Wolfram!

Ignoring the integration constant & assuming $\beta_r > 0$,

it renders: $F(\theta, \beta_r) = \frac{\beta_r\cos(\theta) - \ln\left(\sqrt{2}\beta_r\cos(\theta) + \sqrt{2\left(1+\beta_r^2(\cos^2(\theta)-1)\right)}\right)}{\beta_r}$

and then:

$$F(\pi, \beta_r) - F(0, \beta_r)$$

$$= \frac{\beta_r\cos(\pi) - \ln\left(\sqrt{2}\beta_r\cos(\pi) + \sqrt{2\left(1+\beta_r^2(\cos^2(\pi)-1)\right)}\right)}{\beta_r}$$

$$- \frac{\beta_r\cos(0) - \ln\left(\sqrt{2}\beta_r\cos(0) + \sqrt{2\left(1+\beta_r^2(\cos^2(0)-1)\right)}\right)}{\beta_r}$$

$$= \frac{-\beta_r - \ln\left(-\sqrt{2}\beta_r + \sqrt{2\left(1+\beta_r^2(1-1)\right)}\right)}{\beta_r} - \frac{\beta_r - \ln\left(\sqrt{2}\beta_r + \sqrt{2\left(1+\beta_r^2(1-1)\right)}\right)}{\beta_r}$$

$$= \frac{-\beta_r - \ln\left(-\sqrt{2}\beta_r + \sqrt{2}\right)}{\beta_r} + \frac{-\beta_r + \ln\left(\sqrt{2}\beta_r + \sqrt{2}\right)}{\beta_r}$$

$$= \frac{-\beta_r - \ln\left(\sqrt{2} - \sqrt{2}\beta_r\right)}{\beta_r} + \frac{-\beta_r + \ln\left(\sqrt{2} + \sqrt{2}\beta_r\right)}{\beta_r}$$

$$= \frac{-\beta_r - \ln\left(\sqrt{2}(1-\beta_r)\right) - \beta_r + \ln\left(\sqrt{2}(1+\beta_r)\right)}{\beta_r}$$

$$= \frac{-2\beta_r + \ln\left(\sqrt{2}(1+\beta_r)\right) - \ln\left(\sqrt{2}(1-\beta_r)\right)}{\beta_r} = \frac{-2\beta_r + \ln\left(\frac{1+\beta_r}{1-\beta_r}\right)}{\beta_r}$$

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$$= \frac{\ln\left(\frac{1+\beta_r}{1-\beta_r}\right)}{\beta_r} - 2 = \frac{\ln\left(\frac{1+\beta_r}{1-\beta_r}\right) - 2\beta_r}{\beta_r} = \frac{\ln\left(\frac{1+\beta_r}{1-\beta_r}\right) - \ln\left(e^{2\beta_r}\right)}{\beta_r}$$

(from now on,

$$= \frac{\ln\left(\frac{1+\beta_r}{1-\beta_r} e^{-2\beta_r}\right)}{\beta_r} = \frac{\ln\left(\frac{1+\frac{\omega r}{c}}{1-\frac{\omega r}{c}} e^{-2\frac{\omega r}{c}}\right)}{\frac{\omega r}{c}} = \frac{c \ln\left(\frac{c+\omega r}{c-\omega r} e^{-2\omega r/c}\right)}{\omega r}$$

e = Euler's no.)

We now have: $E_k = 2\pi\rho c^2 \int_0^R r^2 \frac{c \ln\left(\frac{c+\omega r}{c-\omega r} e^{-2\omega r/c}\right)}{\omega r} dr$

which is: $E_k = \frac{2\pi\rho c^3}{\omega} \int_0^R r \ln\left(\frac{c+\omega r}{c-\omega r} e^{-2\omega r/c}\right) dr$

Same story about WolframAlpha ,

yielding: $F(r) = \frac{r}{6}\left(\frac{2\omega r^2}{c} + 3r \ln\left(\frac{c+\omega r}{c-\omega r} e^{-2\omega r/c}\right) + \frac{6c}{\omega}\right) - \frac{c^2 \operatorname{artanh}\frac{\omega r}{c}}{\omega^2}$

which is: $F(r) = \frac{\omega r^3}{3c} + \frac{r^2}{2} \ln\left(\frac{c+\omega r}{c-\omega r} e^{-2\omega r/c}\right) + \frac{rc}{\omega} - \frac{c^2 \operatorname{artanh}\frac{\omega r}{c}}{\omega^2}$

We easily see: $F(0) = 0$

hence: $F(R) - F(0) = F(R)$

$$= \frac{\omega R^3}{3c} + \frac{R^2}{2} \ln\left(\frac{c+\omega R}{c-\omega R} e^{-2\omega R/c}\right) + \frac{cR}{\omega} - \frac{c^2 \operatorname{artanh}\frac{\omega R}{c}}{\omega^2}$$

yielding: $E_k = \frac{2\pi\rho c^3}{\omega}\left(\frac{\omega R^3}{3c} + \frac{R^2}{2} \ln\left(\frac{c+\omega R}{c-\omega R} e^{-2\omega R/c}\right) + \frac{cR}{\omega} - \frac{c^2 \operatorname{artanh}\frac{\omega R}{c}}{\omega^2}\right)$

i.e.: $E_k = 2\pi\rho c^3\left(\frac{R^3}{3c} + \frac{R^2}{2\omega} \ln\left(\frac{c+\omega R}{c-\omega R} e^{-2\omega R/c}\right) + \frac{cR}{\omega^2} - \frac{c^2 \operatorname{artanh}\frac{\omega R}{c}}{\omega^3}\right)$

We also have: $\rho = \frac{3M}{4\pi R^3}$

so it becomes: $E_k = \frac{3Mc^3}{2R^3}\left(\frac{R^3}{3c} + \frac{R^2}{2\omega} \ln\left(\frac{c+\omega R}{c-\omega R} e^{-2\omega R/c}\right) + \frac{cR}{\omega^2} - \frac{c^2 \operatorname{artanh}\frac{\omega R}{c}}{\omega^3}\right)$

therefore: $E_k = \frac{3}{2}Mc^3\left(\frac{1}{3c} + \frac{1}{2\omega R} \ln\left(\frac{c+\omega R}{c-\omega R} e^{-2\omega R/c}\right) + \frac{c}{\omega^2 R^2} - \frac{c^2 \operatorname{artanh}\frac{\omega R}{c}}{\omega^3 R^3}\right)$

With: $\beta_R = \frac{\omega R}{c}$

it turns into: $E_k = \frac{3}{2}Mc^3\left(\frac{1}{3c} + \frac{1}{2c\beta_R} \ln\left(\frac{1+\beta_R}{1-\beta_R} e^{-2\beta_R}\right) + \frac{1}{c\beta_R^2} - \frac{\operatorname{artanh}\beta_R}{c\beta_R^3}\right)$

which is: $E_k = Mc^2\left(\frac{1}{2} + \frac{3}{4\beta_R} \ln\left(\frac{1+\beta_R}{1-\beta_R} e^{-2\beta_R}\right) + \frac{3}{2\beta_R^2} - \frac{3 \operatorname{artanh}\beta_R}{2\beta_R^3}\right)$

For the parenthesised part hereof,
WolframAlpha gives a nicely *sub*luminal
domain of: $-1 < (\beta_R \neq 0) < +1$
and since: $\lim_{\beta_R \to 0} E_k = 0$,
we can safely define: $E_k(\beta_R = 0) \coloneqq 0$
turning it into a continuous
function with domain: $|\beta_R| < 1$

We can replace: $\operatorname{artanh}\beta_R = \frac{1}{2}\ln\left(\frac{1+\beta_R}{1-\beta_R}\right)$

yielding: $E_k = Mc^2\left(\frac{1}{2} + \frac{3}{4\beta_R} \ln\left(\frac{1+\beta_R}{1-\beta_R} e^{-2\beta_R}\right) + \frac{3}{2\beta_R^2} - \frac{3\ln\left(\frac{1+\beta_R}{1-\beta_R}\right)}{4\beta_R^3}\right)$

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i.e.: $$E_\mathrm{k} = Mc^2\left(\frac{1}{2}+\frac{3}{2\beta_R^2}+\frac{3\beta_R^2}{4\beta_R^3}\ln\left(\frac{1+\beta_R}{1-\beta_R}e^{-2\beta_R}\right)-\frac{3\ln\left(\frac{1+\beta_R}{1-\beta_R}\right)}{4\beta_R^3}\right)$$

hence: $$E_\mathrm{k} = Mc^2\left(\frac{1}{2}+\frac{3}{2\beta_R^2}+\frac{3}{4\beta_R^3}\left[\beta_R^2\ln\left(\frac{1+\beta_R}{1-\beta_R}e^{-2\beta_R}\right)-\ln\left(\frac{1+\beta_R}{1-\beta_R}\right)\right]\right)$$

so: $$E_\mathrm{k} = Mc^2\left(\frac{1}{2}+\frac{3}{2\beta_R^2}+\frac{3}{4\beta_R^3}\left[\ln\left(\frac{1+\beta_R}{1-\beta_R}e^{-2\beta_R}\right)^{\beta_R^2}-\ln\left(\frac{1+\beta_R}{1-\beta_R}\right)\right]\right)$$

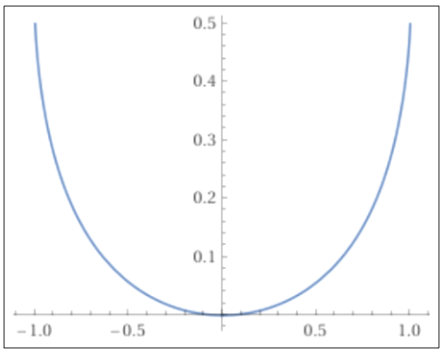
yielding: $$E_\mathrm{k} = Mc^2\left(\frac{1}{2}+\frac{3}{2\beta_R^2}+\frac{3}{4\beta_R^3}\ln\frac{\left(\frac{1+\beta_R}{1-\beta_R}e^{-2\beta_R}\right)^{\beta_R^2}}{\left(\frac{1+\beta_R}{1-\beta_R}\right)}\right)$$

therefore: $$E_\mathrm{k} = Mc^2\left(\frac{1}{2}+\frac{3}{2\beta_R^2}+\frac{3}{4\beta_R^3}\ln\frac{\left(\frac{1+\beta_R}{1-\beta_R}\right)^{\beta_R^2}\left(e^{-2\beta_R}\right)^{\beta_R^2}}{\left(\frac{1+\beta_R}{1-\beta_R}\right)}\right)$$

and, finally: $$\boldsymbol{E_\mathrm{k} = Mc^2\left(\frac{1}{2}+\frac{3}{2\beta_R^2}+\frac{3}{4\beta_R^3}\ln\left(\left(\frac{1+\beta_R}{1-\beta_R}\right)^{(\beta_R^2-1)}e^{-2\beta_R^3}\right)\right)}$$

or: $$\boldsymbol{E_\mathrm{k} = Mc^2\left(\frac{1}{2}+\frac{3}{2\beta_R^2}+\frac{3}{4\beta_R^3}\ln\left(\left(\frac{1-\beta_R}{1+\beta_R}\right)^{(1-\beta_R^2)}e^{-2\beta_R^3}\right)\right)}$$

Plot of parenthesises part:



(from WolframAlpha)

Taylor series: $$\frac{\beta_R^2}{5}+\frac{3\beta_R^4}{35}+\frac{\beta_R^6}{21}+\frac{\beta_R^8}{33}+\mathcal{O}(\beta_R^{10})$$

For non-relativistic equator velocities,
we take the lowest-order term only,

rendering: $$E_\mathrm{k,class} = \frac{1}{5}Mc^2\beta_R^2 = \frac{1}{5}Mv_R^2 = \frac{1}{5}MR^2\omega^2$$

We also obtain: $$E_\mathrm{k}\left(\boldsymbol{\beta_R}=\frac{1}{2}\right) = m_ec^2\left(\frac{1}{2}+\frac{3\cdot4}{2}+\frac{3\cdot8}{4}\ln\left(\left(\frac{1-\frac{1}{2}}{1+\frac{1}{2}}\right)^{\left(1-\frac{1}{4}\right)}e^{-2/8}\right)\right)$$

$$= m_ec^2\left(\frac{13}{2}+6\ln\left(\left(\frac{1}{3}\right)^{\left(\frac{3}{4}\right)}e^{-1/4}\right)\right)$$

$$= m_ec^2\left(\frac{13}{2}+6\ln\left(3^{-3/4}e^{-1/4}\right)\right) = m_ec^2\left(\frac{13}{2}+6\ln\left((27e)^{-1/4}\right)\right)$$

$$= m_ec^2\left(\frac{13}{2}-\frac{6}{4}\ln(27e)\right) = \frac{1}{2}m_ec^2\left(13-3(\ln e+\ln 27)\right)$$

$$= \frac{1}{2}m_ec^2\left(13-3(1+3\ln 2)\right) = \frac{1}{2}m_ec^2(10-3\ln 2)$$

$$= m_ec^2\left(5-\frac{3}{2}\ln 2\right) \approx \mathbf{3.96}m_ec^2 \approx 3.24\times10^{-13}\ \mathrm{J} \approx 2.024\ \mathrm{MeV}$$

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Hmm... now it seems this electron model of mine just collapsed.
This "spinnergy" is roughly *four* times the electron's rest energy.
Shouldn't we measure that as a mass on top of the rest mass?

Second thought:
Of course this spinnergy mass equivalent is
already included in the measued electron mass!
Let's denote the non-spinning bare mass as: m_e^* ,
then: $m_e = m_e^* + m_e^*\left(5-\frac{3}{2}\ln 2\right) = m_e^*\left(6-\frac{3}{2}\ln 2\right)$,
yielding: $m_e^* = m_e/\left(6-\frac{3}{2}\ln 2\right) \approx 0.2016\, m_e$
& then the spinnergy would be: $m_e^* c^2\left(5-\frac{3}{2}\ln 2\right)$
$= m_e c^2 \frac{10-3\ln 2}{12-3\ln 2} \approx 0.7984 m_e c^2 \approx \mathbf{6.5366\times 10^{-14}\ J} \approx \mathbf{0.4080\ MeV}$

Would this spinnergy be totally emitted as a photon,
it would have a frequency of: $\nu = \frac{m_e c^2}{h}\frac{10-3\ln 2}{12-3\ln 2} \approx 9.8649\times 10^{19}$ Hz
and an EM wavelength of: $\lambda = \frac{c}{\nu} \approx 3.0390$ pm
which definitely is a gamma photon.

N.B.: Photons are ***not*** *waves*! Therefore, they do ***not*** have a wavelength!
Only ***waves*** have a wavelength! A photon is an ***energy quantum*** with
a corresponding electromagnetic frequency according to $\boldsymbol{\nu = E/h}$,
but ***NO*** wavelength! As a particle, it flies at the very *speed of light*,
so to us, it is Lorentz contracted to nought point nought, diddly squat.
In its flying direction, it *cannot* have *any* kind of length whatsoever!

We also find: $\frac{\lambda}{\lambda_{C,e}} = \frac{c/\nu}{h/m_e c} = \frac{\frac{c}{\frac{m_e c^2}{h}\frac{10-3\ln 2}{12-3\ln 2}}\cdot m_e c}{h} = \frac{12-3\ln 2}{10-3\ln 2} \approx 1.2525$

For Bremsstrahlung , we consult WikipediA ,
which gives an emitted power of: $P_{a\perp v} = \frac{2q^2a^2\gamma^4}{3c^3} = \frac{2q^2a^2}{3c^3(1-\beta^2)^2}$
which has dimension: $[I^2]\cdot[T]\cdot[L^{-1}]$
so this is definitely ***NOT*** a power, which has: $[M]\cdot[L^2]\cdot[T^{-3}]$

Thank you, WikipediA!

But, luckily, they refer to: *A Plasma Formulary for Physics, Technology, and Astrophysics*, by Declan A. Diver, pp. 46–48.
and I'm glad I found that on Google Books .

It gives (eqn. 4.27 @p.48): $P = \frac{q^2}{6\pi\varepsilon_0 c}\frac{1}{(1-\beta^2)^2}\left(\left|\dot{\vec{\beta}}\right|^2 + \frac{\left(\vec{\beta}\cdot\dot{\vec{\beta}}\right)^2}{1-\beta^2}\right)$

Alas, it also gives a version thereof in CGS units. Oh, you SILLY STUBBORN YANKS! This probably ***is*** the cause of the *misleading malarkey* on WikipediA. One should use ***only*** the **S.I.**, which was designed to be fully coherent & it was already introduced back in 1960, i.e. 65 year ago! Diver's book was published in 2001, i.e. 41 years later. The **Mars Climate Orbiter** disaster (by mixing S.I. & imperial units, even worse) had already happened & what have you learned? MORONS! Misleading, cofnisnug & incoherent unfathomable unit systems should be **ditched**!

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For $\vec{\beta}\cdot\dot{\vec{\beta}}=0$, we get: $P_{a\perp v}=\frac{q^2}{6\pi\varepsilon_0 c}\frac{\left|\dot{\vec{\beta}}\right|^2}{(1-\beta^2)^2}=\frac{q^2a^2}{6\pi\varepsilon_0 c^3(1-\beta^2)^2}=\frac{\mu_0 q^2a^2}{6\pi c(1-\beta^2)^2}$

I can now only ***estimate*** & I ***choose*** (in fact, it hasn't homogenous a & β)

the elementary charge: $q=e$

and the equator velocity: $\beta=\beta_R=\frac{1}{2}$

We had also found: $a_{\mathrm{cp,spin}}=\frac{m_e c^3}{h}\frac{2\sqrt{2}\sqrt{5}\pi}{27\sqrt{3}}\therefore a_{\mathrm{cp,spin}}^2=\frac{2^3 5\pi}{3^7}\frac{m_e^2c^6}{h^2}$

which must be squared,

rendering: $\boldsymbol{P_{a\perp v}}=\frac{\mu_0 e^2}{6\pi c\left(1-\beta_R^2\right)^2}\frac{2^3 5\pi}{3^7}\frac{m_e^2c^6}{h^2}=\frac{\mu_0 e^2}{\frac{9}{16}}\frac{2^2 5}{3^8}\frac{m_e^2c^5}{h^2}$

$=\frac{2^6 5}{3^{10}}\frac{\mu_0 e^2 m_e^2 c^5}{h^2}\approx \mathbf{400.04\ W}$ **WOW!**

One full rotation (ignoring belt trick stuff)

would take: $T=\frac{2\pi R_*\lambda_{C,e}}{\beta_R c}=\frac{2\pi\cdot\frac{\sqrt{3}\left(5-2\beta_R^2\right)^{3/2}}{8\sqrt{5}\pi\beta_R}\cdot h}{\beta_R c\cdot m_e c}=\frac{2\pi\sqrt{3}\left(5-2\beta_R^2\right)^{3/2}h}{8\sqrt{5}\pi\beta_R\beta_R m_e c^2}=\frac{\sqrt{3}\left(5-2\beta_R^2\right)^{3/2}}{4\sqrt{5}\beta_R^2}\frac{h}{m_e c^2}$

($\beta_R=\frac{1}{2}$) yields: $T=\frac{\sqrt{3}\left(\frac{3^2}{2}\right)^{3/2}}{\sqrt{5}}\frac{h}{m_e c^2}=\frac{27\sqrt{3}}{2\sqrt{2}\sqrt{5}}\frac{h}{m_e c^2}\approx \mathbf{5.9844\times10^{-20}\ s}$

The emitted energy

would then be: $\boldsymbol{E_{\mathrm{em/rot}}}=\left(P_{a\perp v}=\frac{2^6 5}{3^{10}}\frac{\mu_0 e^2 m_e^2 c^5}{h^2}\right)\left(T=\frac{27\sqrt{3}}{2\sqrt{2}\sqrt{5}}\frac{h}{m_e c^2}\right)$

$=\frac{2^5 5\sqrt{3}}{3^7\cdot\sqrt{2}\sqrt{5}}\frac{\mu_0 e^2 m_e c^3}{h}\approx \mathbf{4.788\times10^{-17}\ J}$

Now we divide the spinnergy by this emitted power,
yielding an upper limit to the time it could radiate:

$$\Delta\boldsymbol{t_{\max}}=\frac{m_e c^2\frac{10-3\ln 2}{12-3\ln 2}}{\frac{2^6 5}{3^{10}}\frac{\mu_0 e^2 m_e^2 c^5}{h^2}}=\frac{3^{10}(10-3\ln 2)}{2^6 5(12-3\ln 2)}\frac{h^2}{\mu_0 e^2 m_e c^3}\approx \mathbf{8.170\times10^{-17}\ s}$$

The corresponding no. of rotations would be:

$$\boldsymbol{n_{\mathrm{rot}}}=\frac{\Delta t_{\max}}{T}=\frac{3^{10}(10-3\ln 2)}{2^6 5(12-3\ln 2)}\frac{h^2}{\mu_0 e^2 m_e c^3}\frac{2\sqrt{2}\sqrt{5}}{27\sqrt{3}}\frac{m_e c^2}{h}$$

$$=\frac{3^{10}(10-3\ln 2)}{2^6 5(12-3\ln 2)}\frac{2\sqrt{2}\sqrt{5}}{27\sqrt{3}}\frac{h}{\mu_0 e^2 c}=\frac{3^7(10-3\ln 2)\sqrt{2}\sqrt{5}}{2^5 5(12-3\ln 2)\sqrt{3}}\frac{h}{\mu_0 e^2 c}\approx \mathbf{1365}$$

Calculating all of this was a nice exercise that gave me quite some insight,
but the reality is that electrons do not suicidally radiate at 400 W.

It should be obvious that I have not done anything sophisticated as far as quantum mechanics is concerned, except the spin angular momentum, so I am open for refinements in that regard, but I tend to reject all kinds of (implicit) assumptions. And yes, I know about Dirac's belt trick etc. It needs two rotations to return to its original state. However, I wanted to focus on the angular momentum and other simple stuff directly related to any spinning object.

I will now check some other thinkable electron shapes.
Please note:
I do *not* claim an electron would truly be any of them, nor the above!

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Suppose the electron is a homogeneous hollow sphere, a balloon so to say.

Once again, we'll ignore its charge & we must determine its relativistic angular momentum, for which I did not find an out-of-the-box equation, so I'll have to derive it myself. We'll first find the surface of a sphere (cf. the earth!) by integration.

We define: $\theta :=$ colatitude or polar angle (from North Pole along meridian)
and: $\varphi :=$ longitude (from Greenwich meridian eastward along equator)
& we have: $d^2A = du\,dv$ (prod. of locally perpend. infinitesimal displacements)
where: $du =$ *along a circle of latitude* $= r\sin\theta\,d\varphi$
and: $dv =$ *along a meridian* $= r d\theta$
which renders: $d^2A = (r\sin\theta\,d\varphi)(r d\theta) = r^2\sin\theta\,d\varphi d\theta$
hence: $A = \iint d^2A = \int_0^{\pi}\int_0^{2\pi} r^2\sin\theta\,d\varphi\,d\theta$
i.e.: $A = r^2\int_0^{\pi}\sin\theta\int_0^{2\pi} d\varphi\,d\theta$
hence: $A = r^2\int_0^{\pi}\sin\theta\left(\int_0^{2\pi} d\varphi = 2\pi\right)d\theta$
so: $A = 2\pi r^2\int_0^{\pi}\sin\theta\,d\theta = 2\pi r^2\{(-\cos\theta)]_0^{\pi} = \cos\theta]_{\pi}^{0}\}$
yielding: $\boldsymbol{A = 2\pi r^2\big(1-(-1)\big) = 4\pi r^2}$ 👏

Just for fun & recognition, we now treat this surface as *a shell with thickness* dr and integrate this shell's volume: $dV = A dr$ over r from nought to R,
yielding: $\boldsymbol{V = \int_0^R 4\pi r^2 dr = \frac{4}{3}\pi r^3\Big]_0^R = \frac{4}{3}\pi R^3}$ 👌

Similar to A: $Q = \int_0^{\pi}\int_0^{2\pi} q(r,\theta,\varphi)r^2\sin\theta\,d\varphi\,d\theta$
where: $q(r,\theta,\varphi) =$ surface density of *some* quantity Q,
such that: $d^2Q = q(r,\theta,\varphi)r^2\sin\theta\,d\varphi d\theta$

You can freely pull r^2 out of either integral and $\sin\theta$ out of the inner (which integrates only over φ), which may or may not make things easier, depending on the actual form of $q(r,\theta,\varphi)r^2\sin\theta$.

For a homogeneous mass, (with homogeneous *surface* density, that is)
we have: $d^2M = \rho r^2\sin\theta\,d\varphi d\theta$
where: $\rho =$ the *surface* density with dimension [M]·[L^{-2}]
We easily find: $\rho = \frac{M}{4\pi r^2}$ (it's homogeneous & we just calc'd A)
so we obtain: $d^2M = \frac{M}{4\pi}\sin\theta\,d\varphi d\theta$
hence: $\boldsymbol{M = \frac{M}{4\pi}\int_0^{\pi}\sin\theta\int_0^{2\pi} d\varphi\,d\theta = M}$

which is a nice example of kicking in an open door by cyclic reasoning... 😎

In case of a sphere with radius r, spinning along its z-axis,
the velocity of any spot is: $v = \omega r\sin\theta$
& the momentum equals: $d^2p = v d^2M = \omega r\sin\theta\,d^2M = \omega\rho r^3\sin^2\theta\,d\varphi d\theta$
& the angular momentum: $d^2L = r\sin\theta\,d^2p = \omega\rho r^4\sin^3\theta\,d\varphi d\theta$

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Relativistic ang. mom.: $d^2L_{\text{rel}} = \gamma d^2L = \frac{\omega\rho r^4 \sin^3\theta}{\sqrt{1-\frac{v^2}{c^2}}} d\varphi d\theta = \frac{\omega\rho r^4 \sin^3\theta}{\sqrt{1-\frac{(\omega r \sin\theta)^2}{c^2}}} d\varphi d\theta$

hence: $L_{\text{rel}} = \int_0^{\pi}\int_0^{2\pi} \frac{\omega\rho r^4 \sin^3\theta}{\sqrt{1-\frac{\omega^2 r^2 \sin^2\theta}{c^2}}} d\varphi\, d\theta$

i.e.: $L_{\text{rel}} = \int_0^{\pi} \frac{\omega\rho r^4 \sin^3\theta}{\sqrt{1-\frac{\omega^2 r^2 \sin^2\theta}{c^2}}} \int_0^{2\pi} d\varphi\, d\theta$

With: $\beta_r := \frac{\omega r}{c}$ = equator velocity

it becomes: $L_{\text{rel}} = 2\pi\omega\rho r^4 \int_0^{\pi} \frac{\sin^3\theta}{\sqrt{1-\beta_r^2 \sin^2\theta}} d\theta$

And once again, WolframAlpha ***does*** come up with the ***in***definite integral,
but it apparently doesn't know how to subtract its boundary values 😟.
Zero points earned, Mr. Wolfram!

After some rewriting by myself and restricting it to $\beta_r \geq 0$,

it gives:
$$\int \frac{\sin^3\theta}{\sqrt{1-\beta_r^2\sin^2\theta}} d\theta$$
$$= \frac{\cos(\theta)\sqrt{\beta_r^2(\cos(2\theta)-1)+2}}{2\sqrt{2}\beta_r^2} - \frac{(\beta_r^2+1)\ln\left(\sqrt{2\beta_r^2}\cos(\theta)+\sqrt{\beta_r^2(\cos(2\theta)-1)+2}\right)}{2\beta_r^3} + C$$
$$=: F(\theta,\beta_r) + C$$

So we have to calculate:

$$F(\boldsymbol{\pi},\beta_r) = \frac{\cos(\pi)\sqrt{\beta_r^2(\cos(2\pi)-1)+2}}{2\sqrt{2}\beta_r^2} - \frac{(\beta_r^2+1)\ln\left(\sqrt{2\beta_r^2}\cos(\pi)+\sqrt{\beta_r^2(\cos(2\pi)-1)+2}\right)}{2\beta_r^3}$$
$$= \frac{(-1)\sqrt{\beta_r^2(1-1)+2}}{2\sqrt{2}\beta_r^2} - \frac{(\beta_r^2+1)\ln\left(\sqrt{2\beta_r^2}(-1)+\sqrt{\beta_r^2(1-1)+2}\right)}{2\beta_r^3}$$
$$= \frac{-1}{2\beta_r^2} - \frac{(\beta_r^2+1)\ln\left(\sqrt{2}-\sqrt{2\beta_r^2}\right)}{2\beta_r^3}$$
$$= \frac{-1}{2\beta_r^2} - \frac{(\beta_r^2+1)\ln\left(\sqrt{2}(1-\beta_r)\right)}{2\beta_r^3}$$

and:
$$F(\boldsymbol{0},\beta_r) = \frac{\cos(0)\sqrt{\beta_r^2(\cos(0)-1)+2}}{2\sqrt{2}\beta_r^2} - \frac{(\beta_r^2+1)\ln\left(\sqrt{2\beta_r^2}\cos(0)+\sqrt{\beta_r^2(\cos(0)-1)+2}\right)}{2\beta_r^3}$$
$$= \frac{\sqrt{\beta_r^2(1-1)+2}}{2\sqrt{2}\beta_r^2} - \frac{(\beta_r^2+1)\ln\left(\sqrt{2\beta_r^2}+\sqrt{\beta_r^2(1-1)+2}\right)}{2\beta_r^3}$$
$$= \frac{1}{2\beta_r^2} - \frac{(\beta_r^2+1)\ln\left(\sqrt{2}(1+\beta_r)\right)}{2\beta_r^3}$$

and then: $L_{\text{rel}} = 2\pi\omega\rho r^4\left(F(\boldsymbol{\pi},\beta_r) - F(\boldsymbol{0},\beta_r)\right)$

hence: $L_{\text{rel}} = 2\pi\omega\rho r^4\left(-\frac{1}{2\beta_r^2} - \frac{(\beta_r^2+1)\ln\left(\sqrt{2}(1-\beta_r)\right)}{2\beta_r^3} - \frac{1}{2\beta_r^2} + \frac{(\beta_r^2+1)\ln\left(\sqrt{2}(1+\beta_r)\right)}{2\beta_r^3}\right)$

i.e.: $L_{\text{rel}} = 2\pi\omega\rho r^4\left(\frac{(\beta_r^2+1)\ln\left(\sqrt{2}(1+\beta_r)\right)}{2\beta_r^3} - \frac{(\beta_r^2+1)\ln\left(\sqrt{2}(1-\beta_r)\right)}{2\beta_r^3} - \frac{1}{\beta_r^2}\right)$

so: $L_{\text{rel}} = 2\pi\omega\rho r^4\left(\frac{(\beta_r^2+1)\ln\left(\sqrt{2}(1+\beta_r)\right)}{2\beta_r^3} - \frac{(\beta_r^2+1)\ln\left(\sqrt{2}(1-\beta_r)\right)}{2\beta_r^3} - \frac{2\beta_r}{2\beta_r^3}\right)$

yielding: $L_{\text{rel}} = 2\pi\omega\rho r^4 \frac{(\beta_r^2+1)\ln\left(\sqrt{2}(1+\beta_r)\right) - (\beta_r^2+1)\ln\left(\sqrt{2}(1-\beta_r)\right) - 2\beta_r}{2\beta_r^3}$

which is: $L_{\text{rel}} = \pi\omega\rho r^4 \frac{(\beta_r^2+1)\left(\ln\left(\sqrt{2}(1+\beta_r)\right) - \ln\left(\sqrt{2}(1-\beta_r)\right)\right) - 2\beta_r}{\beta_r^3} = \pi\boldsymbol{\omega}\rho\boldsymbol{r^4}\frac{(1+\beta_r^2)\ln\left(\frac{1+\beta_r}{1-\beta_r}\right) - 2\beta_r}{\beta_r^3}$

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so: $L_{\text{rel}} = \pi\boldsymbol{\omega r}\rho\boldsymbol{r^3}\frac{(1+\beta_r^2)\ln\left(\frac{1+\beta_r}{1-\beta_r}\right)-2\beta_r}{\beta_r^3} = \pi\boldsymbol{\beta_r c}\rho r^3\frac{(1+\beta_r^2)\ln\left(\frac{1+\beta_r}{1-\beta_r}\right)-2\beta_r}{\boldsymbol{\beta_r^3}}$

therefore: $L_{\text{rel}} = \pi c\rho r^3\frac{(1+\beta_r^2)\ln\left(\frac{1+\beta_r}{1-\beta_r}\right)-2\beta_r}{\beta_r^2}$

This is an odd function: $L_{\text{rel}}(-\beta_r) = -L_{\text{rel}}(\beta_r)$

so the aforementioned restriction of: $\beta_r \geq 0$

seems no longer necessary.

For $\beta_r = 0$ however, it renders: $\frac{0}{0} = \boldsymbol{?}$

but on WolframAlpha we see that defining: $L_{\text{rel}}(\beta_r = 0) := 0$

turns it into a continuous function without that infinitesimally small hole[4] at the origin.

Doing so (& avoiding complex numbers)

yields a domain of: $|\beta_r \in \mathbb{R}| < 1$

so a **presumed** hollow electron's equator would never exceed the *speed of light*!

The series at $\beta_r = 0$, (div'd by $\pi c\rho r^3$): $\sum_{n=0}^{\infty}\frac{8(n+1)\beta_r^{2n+1}}{(2n+1)(2n+3)}$ (see Appendix I @p.15)

$= \frac{8\beta_r}{3} + \frac{16\beta_r^3}{15} + \frac{24\beta_r^5}{35} + \mathcal{O}(\beta_r^7)$

also yields: $L_{\text{rel}}(\beta_r = 0) = 0$

so we can indeed safely fill the hole in order to make it a continuous function.

We substitute: $\rho = \frac{m_e}{4\pi r^2}$ (electron mass, of course)

yielding: $L_{\text{rel}} = \pi c\frac{m_e}{4\pi r^2}r^3\frac{(1+\beta_r^2)\ln\left(\frac{1+\beta_r}{1-\beta_r}\right)-2\beta_r}{\beta_r^2}$

& therefore: $\boldsymbol{L_{\text{rel}}(\beta_r \neq 0) = cm_e r\frac{(1+\beta_r^2)\ln\left(\frac{1+\beta_r}{1-\beta_r}\right)-2\beta_r}{4\beta_r^2}}$, $\boldsymbol{L_{\text{rel}}(\beta_r = 0) = 0}$

This should be the *relativistic angular momentum* of a spinning homogeneous hollow uncharged sphere of mass $\boldsymbol{m_e}$ with radius $\boldsymbol{r}$ and equator velocity $\boldsymbol{\beta_r}$.

We now equate: $\boldsymbol{S = L_{\text{rel}}} \therefore \frac{\sqrt{3}h}{4\pi} = cm_e r\frac{(1+\beta_r^2)\ln\left(\frac{1+\beta_r}{1-\beta_r}\right)-2\beta_r}{4\beta_r^2}$

(I do however not know in how far: $S = \sqrt{s(s+1)}\hbar$
depends on any **presumed** electron shape, so maybe I am now going astray)

which simplifies to: $\frac{h}{cm_e} = \frac{\pi r}{\sqrt{3}}\cdot\frac{(1+\beta_r^2)\ln\left(\frac{1+\beta_r}{1-\beta_r}\right)-2\beta_r}{\beta_r^2}$

i.e.: $\frac{\lambda_{C,e}}{r} = \frac{\pi}{\sqrt{3}}\cdot\frac{(1+\beta_r^2)\ln\left(\frac{1+\beta_r}{1-\beta_r}\right)-2\beta_r}{\beta_r^2}$

or: $\boldsymbol{R_* = \frac{R}{\lambda_{C,e}} = \frac{\sqrt{3}\beta_R^2}{\pi\left((1+\beta_R^2)\ln\left(\frac{1+\beta_R}{1-\beta_R}\right)-2\beta_R\right)}}$ (changed r to R)

WolframAlpha finds: $\boldsymbol{R_* = 1 \Rightarrow \beta_R \approx 0.2033}$

as well as: $\boldsymbol{R_* = \frac{4}{\pi} \Rightarrow \beta_R \approx 0.1607}$

[4] *Als jij gelooft in infinitesimaal kleine gaatjes, geloof ik dat er in je hoofd een groter gaatje zit.*
If you believe in infinitesimally small holes, then I believe you've got a larger little hole in your head
(Dutch idiomatic expression for sillyness, a *little hole in your head* allows all of your wisdom to escape *into* the blue).

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Although both equator velocities are fully plausible and the hollow sphere would never feel the need to breed or seed a deed that exceeds the speed of lɋght,

— Ceterum censeo superluminalitatem esse delendam! Per Jovem & Albertum! —

I prefer the values found for a solid sphere, simply because they surround $\boldsymbol{\beta_R = 1/2}$, which is a value that gives me good feelings.

Let's now suppose the electron is an equatorial disk.

Quid hoc nunc est, uas dinna uere? Hae sae quetocal ni. Nedus, daeis de trucni. *Lorem Ipsum*[5] iam ab annis 1500 textus ficticius typicus industriae fuit: Lorem ipsum dolor sit amet, consectetur adipiscing elit, sed do eiusmod tempor incididunt ut labore et dolore magna aliqua. Ut enim ad minim veniam, quis nostrud exercitation ullamco laboris nisi ut aliquip ex ea commodo consequat. Duis aute irure dolor in reprehenderit in voluptate velit esse cillum dolore eu fugiat nulla pariatur. Excepteur sint occaecat cupidatat non proident, sunt in culpa qui officia deserunt mollit anim id est laborum.

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Let's now suppose the electron merely is a ring, its own equator.

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[5] See: https://www.lipsum.com/

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Et nunc nonnulla ioca linguae: Tete roro mama nunu dada tete lala tete[6]. Mea mater sus est mala[7] (commata magni momenti sunt). Ave, ave, aveo esse aves[8]. Modo aliquid investigavi. Mea culpa, mea maxima culpa. Eo contentus esto.

Orationes finales.

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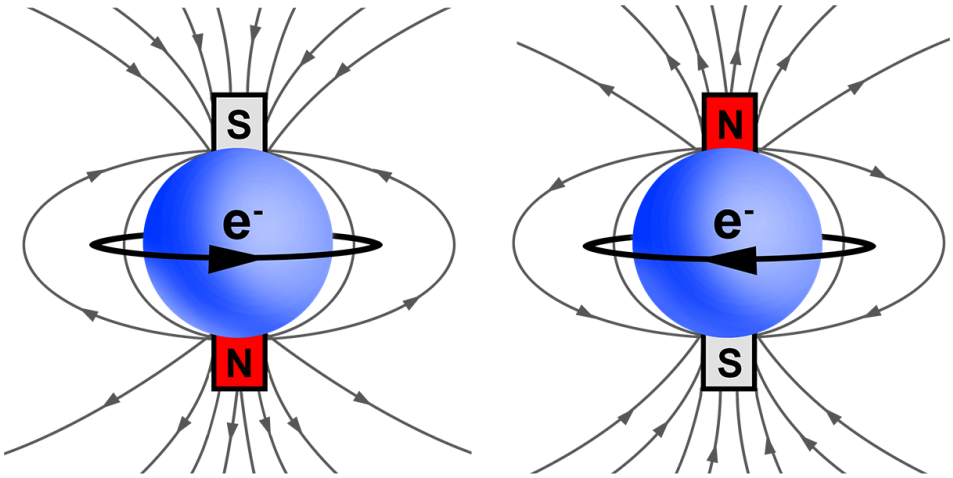
Atque haec est absurdissima ineptias quam umquam Henricus Reintius excogitavit.
Scis, Henkus Tancus Sapiens, qui est optimus in toto mundo. Henk de Denktank, dus.

Henk Reints
Henk-Reints.nl



https://www.trentu.ca/online-legacy/modules/Chem/img/mod2/Orbits_Picture_transparent.png

[6] Te tero Roma, manu nuda; date tela, late te! =
Ik vermorzel je, Rome, met blote hand; geef me je wapens en ga schuilen!
I'll crush you, Rome, with my bare hands; give me your weapons and hide yourself!

[7] *My mother is an evil pig*. Or: Mea, mater, sus est mala. = *Hurry, mother, the pig is eating the apples*.

[8] *Hi, gramps, I want to eat birds*.

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Appendix I

Find series for: $\frac{(1+x^2)\ln\left(\frac{1+x}{1-x}\right)-2x}{x^2}$

McLaurin series of $\ln(1+x) = \sum_{n=1}^{\infty}\frac{-(-x)^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$

hence: $\ln(1-x) = \sum_{n=1}^{\infty}\frac{-x^n}{n} = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \cdots$

so: $\ln\left(\frac{1+x}{1-x}\right) = \ln(1+x) - \ln(1-x) = \sum_{n=1}^{\infty}\left(\frac{-(-x)^n}{n} - \frac{-x^n}{n}\right) = \sum_{n=1}^{\infty}\frac{x^n-(-x)^n}{n}$

$$= \sum_{k=0}^{\infty}\left(\frac{x^{2k+1}-(-x)^{2k+1}}{2k+1} + \frac{x^{2k+2}-(-x)^{2k+2}}{2k+2}\right)$$

$$= \sum_{k=0}^{\infty}\left(\frac{x^{2k+1}-(-1)^{2k+1}x^{2k+1}}{2k+1} + \frac{x^{2k+2}-(-1)^{2k+2}x^{2k+2}}{2k+2}\right)$$

$$= \sum_{k=0}^{\infty}\left(\frac{x^{2k+1}-\left(-x^{2k+1}\right)}{2k+1} + \frac{x^{2k+2}-x^{2k+2}}{2k+2}\right)$$

$$= \sum_{k=0}^{\infty}\frac{2x^{2k+1}}{2k+1}$$

substituting n for k :

$$\frac{(1+x^2)\sum_{n=0}^{\infty}\frac{2x^{2n+1}}{2n+1}-2x}{x^2}$$

$$= \frac{2\left(\sum_{n=0}^{\infty}\frac{(1+x^2)x^{2n+1}}{2n+1}-x\right)}{x^2}$$

$$= 2\left(\sum_{n=0}^{\infty}\frac{(1+x^2)x^{2n+1}}{(2n+1)x^2} - \frac{x}{x^2}\right) = 2\left(\sum_{n=0}^{\infty}\frac{x^{2n+1}+x^2x^{2n+1}}{(2n+1)x^2} - \frac{1}{x}\right)$$

$$= 2\left(\sum_{n=0}^{\infty}\frac{\frac{x^{2n+1}}{x^2}+\frac{x^2x^{2n+1}}{x^2}}{2n+1} - \frac{1}{x}\right) = 2\left(\sum_{n=0}^{\infty}\frac{x^{2n-1}+x^{2n+1}}{2n+1} - \frac{1}{x}\right)$$

$$= 2\left(\frac{x^{2\cdot0-1}+x^{2\cdot0+1}}{2\cdot0+1} + \sum_{n=1}^{\infty}\frac{x^{2n-1}+x^{2n+1}}{2n+1} - \frac{1}{x}\right)$$

$$= 2\left(x^{-1} + x + \sum_{n=1}^{\infty}\frac{x^{2n-1}+x^{2n+1}}{2n+1} - x^{-1}\right)$$

$$= 2\left(x + \sum_{n=1}^{\infty}\frac{x^{2n-1}+x^{2n+1}}{2n+1}\right) = 2\left(x + \sum_{n=1}^{\infty}\frac{x^{2n-1}}{2n+1} + \sum_{n=1}^{\infty}\frac{x^{2n+1}}{2n+1}\right)$$

$$= 2\left(x + \frac{x^{2\cdot1-1}}{2\cdot1+1} + \sum_{n=2}^{\infty}\frac{x^{2n-1}}{2n+1} + \sum_{n=1}^{\infty}\frac{x^{2n+1}}{2n+1}\right) = 2\left(x + \frac{x}{3} + \sum_{n=2}^{\infty}\frac{x^{2n-1}}{2n+1} + \sum_{n=1}^{\infty}\frac{x^{2n+1}}{2n+1}\right)$$

substitute $m = n-1 \therefore n = m+1$:

$$= 2\left(\frac{4x}{3} + \sum_{m=1}^{\infty}\frac{x^{2(m+1)-1}}{2(m+1)+1} + \sum_{n=1}^{\infty}\frac{x^{2n+1}}{2n+1}\right) = 2\left(\frac{4x}{3} + \sum_{m=1}^{\infty}\frac{x^{2m+1}}{2m+3} + \sum_{n=1}^{\infty}\frac{x^{2n+1}}{2n+1}\right)$$

substitute: $n = m$:

$$= 2\left(\frac{4x}{3} + \sum_{n=1}^{\infty}\frac{x^{2n+1}}{2n+3} + \sum_{n=1}^{\infty}\frac{x^{2n+1}}{2n+1}\right) = 2\left(\frac{4x}{3} + \sum_{n=1}^{\infty}\left(\frac{x^{2n+1}}{2n+3} + \frac{x^{2n+1}}{2n+1}\right)\right)$$

$$= 2\left(\frac{4x}{3} + \sum_{n=1}^{\infty}\left(\frac{(2n+1)x^{2n+1}}{(2n+1)(2n+3)} + \frac{(2n+3)x^{2n+1}}{(2n+1)(2n+3)}\right)\right) = 2\left(\frac{4x}{3} + \sum_{n=1}^{\infty}\frac{((2n+1)+(2n+3))x^{2n+1}}{(2n+1)(2n+3)}\right)$$

$$= 2\left(\frac{4x}{3} + \sum_{n=1}^{\infty}\frac{(4n+4)x^{2n+1}}{4n^2+8n+3}\right) = \frac{8x}{3} + \sum_{n=1}^{\infty}\frac{(8n+8)x^{2n+1}}{4n^2+8n+3} = \frac{(8\cdot\mathbf{0}+8)x^{2\cdot\mathbf{0}+1}}{4\cdot\mathbf{0}^2+8\cdot\mathbf{0}+3} + \sum_{n=1}^{\infty}\frac{(8n+8)x^{2n+1}}{4n^2+8n+3}$$

$$= \sum_{n=0}^{\infty}\frac{\mathbf{8(n+1)x^{2n+1}}}{\mathbf{(2n+1)(2n+3)}}$$

👏 *Plaudite, amici, comedia finita est.* 👏

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