We'll restrict ourselves to positive numbers & call the set of all positive rational nos.: $\mathbb{Q}^+$

We erase all natural numbers from it: $\mathbb{Q}^* := \mathbb{Q}^+ \setminus \mathbb{N}$

so we easily find: $(q \in \mathbb{Q}^*) \notin \mathbb{N}$

Of course all these $(q \in \mathbb{Q}^*)$ have been reduced to the ultimate, i.e.: $\gcd(num, den) = 1$

so numerator & denominator have no common factors.

Raising $(q \in \mathbb{Q}^*)$ to $(n \in \mathbb{N})$ obviously does not create new common factors of numerator & denominator.

Therefore, we clearly have: $(q \in \mathbb{Q}^*)^{(n\in\mathbb{N})} \notin \mathbb{N}$

which means: $(q \in \mathbb{Q}^*) = \sqrt[(n\in\mathbb{N})]{(x \notin \mathbb{N})}$

hence, using Modus Tollens (sort of): $\sqrt[(n\in\mathbb{N})]{(m \in \mathbb{N})} \notin \mathbb{Q}^*$

If & only if $(\hat{m} \in \mathbb{N})$ is a perfect n^{th} power: $\left[\hat{m} = (k \in \mathbb{N})^{(n\in\mathbb{N})}\right] \in \mathbb{N}$

we obtain: $\sqrt[(n\in\mathbb{N})]{\hat{m}} \in \mathbb{N}$

We'll denote all other $(m \in \mathbb{N})$ as $\tilde{m}$ & these obviously complement the above: $\sqrt[(n\in\mathbb{N})]{\tilde{m}} \notin \mathbb{N}$

Together with $\notin \mathbb{Q}^*$ as found above, this yields: $\sqrt[(n\in\mathbb{N})]{\tilde{m}} \notin \mathbb{Q}^+$

Any natural root of any natural number is either natural or ***ir***rational.

Of course this includes $\left(\sqrt{2} \notin \mathbb{Q}\right)$, as already found by $\Pi\upsilon\vartheta\alpha\gamma o\rho\alpha\varsigma$.