A ball of close-packed neutronium

Total mass:	M			
neutron mass:	m_n	≈ 1.67492750056	$\times 10^{-27}$	kg
number of neutrons:	N	$= \frac{M}{m_n}$		
neutron radius[1]:	r_n	$\approx 0.\mathbf{84}$	$\times 10^{-15}$	m
neutron volume:	V_n	$= \frac{4\pi}{3} r_n^3$		
		≈ 2.48	$\times 10^{-45}$	m^3
net total volume:	V_{net}	$= NV_n = \frac{4\pi}{3}\frac{M}{m_n} r_n^3$		
Close packing factor:	ξ	$:= \frac{\pi}{3\sqrt{2}}$		
gross total volume:	V_{tot}	$= \frac{1}{\xi}\frac{4\pi}{3}\frac{M r_n^3}{m_n} = \frac{4\sqrt{2} M r_n^3}{m_n}$		
gross total radius:	r_{tot}	$= \sqrt[3]{\frac{3}{4\pi}V_{\text{tot}}} = \sqrt[3]{\frac{3\sqrt{2}}{\pi}\frac{M}{m_n}}\cdot r_n$		
total gravitational potential energy:	$E_{\text{p,g,tot}}$	$= \frac{-3GM^2}{5 r_{\text{tot}}} = \frac{-3GM^2}{5\cdot\sqrt[3]{\frac{3\sqrt{2}}{\pi}}\cdot\frac{M^{1/3}}{m_n^{1/3}}\cdot r_n} = \frac{-3\sqrt[3]{\pi}GM^{5/3}m_n^{1/3}}{5\cdot\sqrt[3]{3\sqrt{2}}\cdot r_n}$		
		$= \frac{-\sqrt[3]{27\pi}GM^{5/3}m_n^{1/3}}{5\cdot\sqrt[3]{3}\sqrt[6]{2}\cdot r_n} = \frac{-\sqrt[3]{9\pi}GM^{5/3}m_n^{1/3}}{5\cdot\sqrt[6]{2} r_n}$		
gravitational potential energy per neutron:	$E_{\text{p,g},n}$	$= \frac{-\sqrt[3]{9\pi}GM^{5/3}m_n^{1/3}}{5\cdot\sqrt[6]{2}\, r_n \boldsymbol{N}} = \frac{-\sqrt[3]{9\pi}}{5\cdot\sqrt[6]{2}}\frac{G m_n^{4/3}}{r_n} M^{2/3}$		
	$\Xi := \frac{-\sqrt[3]{9\pi}}{5\cdot\sqrt[6]{2}}\frac{G m_n^{4/3}}{r_n}$	≈ -8.5791365	$\times 10^{-32}$	$J/(kg^{2/3})$
	$E_{\text{p,g},n}$	$\Xi M^{2/3}$		
Neutron decay energy[2]:	E_β	≈ 0.782343 MeV ≈ 1.25345	$\times 10^{-13}$	J
neutronium would be stable if:	$\lvert E_{\text{p,g},n}\rvert > \lvert E_\beta\rvert$			
tipping point:	$\left(M = \sqrt{\left\lvert\frac{E_{\text{p,g},n}}{\Xi}\right\rvert^3}\right) = \sqrt{\left\lvert\frac{E_\beta}{\Xi}\right\rvert^3}$	≈ 1.766	$\times 10^{27}$	kg
min. mass of stable ball of neutronium:	$\boldsymbol{M_{n,\text{min}}}$	$\boldsymbol{\approx 8.88}$	$\boldsymbol{\times 10^{-4}}$	$\boldsymbol{M_\odot}$
		$\approx \boldsymbol{M_\odot/1100} \approx \boldsymbol{M_{\text{Jup}}} \approx 1.898$	$\times 10^{27}$	kg

This calculation/estimate used classical mechanics only.

Such light neutron stars have a.f.a.i.k. never been observed, so either the circumstances for forming them are in practice never met, or maybe they cannot even exist at all. I haven't yet taken any quantum mechanics into account, especially *degeneracy pressure* (see below). The above should however rather clearly indicate the large gap between atomic nuclei and large balls of neutronium. Intermediate amounts of solid nucleonic matter are impossible.

[1] See https://henk-reints.nl/HR-Existance-postulate.pdf , where I rather elegantly derive this (speculative) value.
[2] From https://en.wikipedia.org/wiki/Free_neutron_decay#Energy_budget as of 2025-08-24

Let's calculate something interesting.

Close-packed neutronium density:	$\boldsymbol{\rho_{n,\mathrm{cp}}}$	$= \frac{M}{V_{\mathrm{tot}}} = \frac{3\xi m_n M}{4\pi M r_n^3} = \frac{3\xi m_n}{4\pi r_n^3} = \frac{3\pi m_n}{4\pi \cdot 3\sqrt{2} r_n^3} = \frac{m_n}{4\sqrt{2} r_n^3}$		
which equals:		≈ 4.996	$\times 10^{17}$	kg/m³
At size of **ping pong ball** ($\varnothing = 40$ mm):	$\boldsymbol{M_{n,\mathrm{pp}}}$	$= \frac{m_n}{4\sqrt{2} r_n^3} \cdot \frac{4\pi}{3} r_{\mathrm{pp}}^3 = \frac{\pi m_n}{3\sqrt{2}} \cdot \left(\frac{r_{\mathrm{pp}}}{r_n}\right)^3$		
which equals:		$\approx \mathbf{1.67}$	$\times \mathbf{10^{13}}$	**kg**

This ping pong ball roughly has one tenth of the mass of Mount Everest!

At its surface, the specific gravitational force would be:	$\boldsymbol{g_{\mathrm{pp}}} = \frac{GM}{r^2}$	$\approx \mathbf{2.79}$	$\times \mathbf{10^6}$	**N/kg**
Estimated mean weight of a **passenger car**:	M_{car}	≈ 1670		kg
yields:	pingpeunium[3] = **ten billion** (NL: tien miljard) **cars!**			
Mean gravitational pressure:	$\overline{\boldsymbol{P_g}} = \frac{E_{\mathrm{p,g,tot}}}{V_{\mathrm{tot}}} = \frac{-\sqrt[3]{9\pi} G M^{5/3} m_n^{1/3}}{5\sqrt[6]{2} r_n} \frac{m_n}{4\sqrt{2} M r_n^3} = \frac{-\sqrt[3]{9\pi} G m_n^{4/3}}{20\sqrt[3]{4} r_n^4} \boldsymbol{M^{2/3}}$			
We find (negative means it is compressive):	$\frac{-\sqrt[3]{9\pi} G m_n^{4/3}}{20\sqrt[3]{4} r_n^4}$	≈ -2.558766945	$\times 10^{13}$	Pa/(kg$^{2/3}$)

Pauli principle: in one quantum system, no two fermions can be in identically the same state. This prevents neutrons to dive to the bottom of the potential well, so they cannot get rid of all their energy, yielding a remaining energy density which is called *degeneracy pressure*. It implies a (not precisely known) maximum to the possible size of a nucleus (heaviest found sofar is $^{294}_{118}$Og (Oganesson), $t_{1/2} \approx 0.7$ ms). Above this maximum, the nucleus simply cannot bind them anymore. Hey, they won't stay all day, but fly away, go astray & decay. Mayday, mayday, mayday[4]. Pray! Until the thing becomes so massive that the gravitional pressure sufficiently exceeds this degeneracy pressure! Then it will — at least in this regard — turn into a stable ball of neutronium. Let's have a look at the degeneracy pressure.

Wikipedia gives (I hate $\hbar$, certainly when m'plied by π):

$$\boldsymbol{P_{\mathrm{deg}}} = \frac{(3\pi^2)^{2/3}\hbar^2}{5m}\left(\frac{N}{V}\right)^{5/3} = \frac{3^{2/3}\pi^{4/3}h^2}{5m\cdot 4\pi^2}\left(\frac{N}{V}\right)^{5/3} = \left(\frac{3}{\pi}\right)^{2/3}\frac{h^2}{20m}\left(\frac{N}{V}\right)^{5/3}$$

electrons in an atom:

$$= \left(\frac{3}{\pi}\right)^{2/3}\frac{h^2}{20m_e}\left(\frac{Z}{V_{at}}\right)^{5/3} = \left(\frac{3}{\pi}\right)^{2/3}\frac{h^2}{20m_e}\left(\frac{3Z}{4\pi r_{at}^3}\right)^{5/3} = \frac{9h^2}{160 m_e r_{at}^5}\left(\frac{3Z^5}{2\pi^7}\right)^{1/3}$$

Example: **water:** $Z_{\mathrm{H_2O}} = 8$, $r_{\mathrm{H_2O}} \approx 0.1925$ nm, $m_e \approx 9.11 \times 10^{-31}$ kg $\Rightarrow$ **~260 GPa = $\mathbf{2.6 \times 10^6}$ bar**

This equation applies to a non-relativistic ideal Fermi gas. But *gas* implies more or less freely moving molecules[5], which is not the case in case of a case of solid neutronium.

[6]gives (non-relativistic):

$$\boldsymbol{P_{\mathrm{NR}}} = \frac{\hbar^2}{5}\left(\frac{3\pi^2}{m_n^4}\right)^{2/3}\rho^{5/3} = \frac{h^2}{5(2\pi)^2}\left(\frac{3\pi^2}{m_n^4}\right)^{2/3}\rho^{5/3} = \frac{h^2}{20}\left(\frac{3}{\pi m_n^4}\right)^{2/3}\rho^{5/3}$$

& ultra-relativistic:

$$\boldsymbol{P_{\mathrm{UR}}} = \frac{\hbar c}{4}\left(\frac{3\pi^2}{m_n^4}\right)^{2/3}\rho^{4/3} = \frac{5\pi c\rho^{-1/3}}{2h}P_{\mathrm{NR}} = \frac{\pi h c}{8}\left(\frac{3}{\pi m_n^4}\right)^{2/3}\rho^{4/3}$$

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[3] Graag op z'n Nederlands uitspreken! Sorry if you don't grasp this. In Dutch, pingpeunium sounds a bit funny. But to most native English speakers, it may be rather hard to properly pronounce the Dutch *eu* sound.

[4] I've once learned that, over a century ago, some English pilot got into trouble while flying over France. *Paris, we have a problem*. Where normal people would call: *Help!*, an Englishman kindly asks: *Dear sir, I think you are certainly aware of my troublesome situation and you seem very able to see I'm obviously having a slight problem that I cannot handle by myself, so could you please give me some assistence in trying to survive?*. The French tend to say: *Au secours!* He didn't know that. But: *Aidez moi!* would have worked just as good. Neither that was part of his knowledge. Instead, he called through his radio: *M'aidez!* English speakers pronounce that: *Mayday!*, thereby stressing the first syllable, which is rather ***un***French. Hence, the French didn't grasp it quickly enough. May his soul rest in pieces... This would be how *Mayday* became an emergency call. It should be said three times in a row & don't you *dare* to say it as a joke while you're actually flying a plane or so!

[5] Molecules in a physical sense, not chemical.

[6] https://homes.psd.uchicago.edu/~sethi/Teaching/P243-W2020/final-papers/helstrom.pdf , §2, eqns. 2.39 & 2.40.

We check:

$$P_{\text{deg}} = \left(\frac{3}{\pi}\right)^{2/3} \frac{h^2}{20m} \left(\frac{N}{V}\right)^{5/3} = \left(\frac{3}{\pi}\right)^{2/3} \frac{h^2}{20m_n} \left(\frac{M}{Vm_n}\right)^{5/3} = \left(\frac{3}{\pi}\right)^{2/3} \frac{h^2}{20} \frac{1}{m_n^{8/3}} \rho^{5/3}$$

$$P_{\text{NR}} = \frac{h^2}{20} \left(\frac{3}{\pi m_n^4}\right)^{2/3} \rho^{5/3} = \left(\frac{3}{\pi}\right)^{2/3} \frac{h^2}{20} \frac{1}{m_n^{8/3}} \rho^{5/3}$$

Hence, both sources indeed give the same equation, albeit it in different forms.

We now choose next form: $P_{\text{deg}} = \left(\frac{3}{\pi}\right)^{2/3} \frac{h^2}{20m_n^{8/3}} \frac{M^{5/3}}{V^{5/3}}$

and substitute the found: $V = V_{\text{tot}} = \frac{4\sqrt{2}Mr_n^3}{m_n}$

yielding: $P_{\text{deg}} = \left(\frac{3}{\pi}\right)^{2/3} \frac{h^2}{20m_n^{8/3}} \left(\frac{m_n}{4\sqrt{2}Mr_n^3}\right)^{5/3} M^{5/3}$

which simplifies to: $P_{\text{deg}} = \left(\frac{3}{\pi}\right)^{2/3} \frac{1}{5\cdot 2^{37/6}} \frac{h^2}{m_n r_n^5}$

Now we add this to: $\overline{P}_{\text{g}} = \frac{-\sqrt[3]{9\pi}Gm_n^{4/3}}{20\sqrt[3]{4}r_n^4} M^{2/3}$

resulting in: $P = \left(\frac{3}{\pi}\right)^{2/3} \frac{1}{5\cdot 2^{37/6}} \frac{h^2}{m_n r_n^5} - \frac{\sqrt[3]{9\pi}Gm_n^{4/3}}{20\sqrt[3]{4}r_n^4} M^{2/3}$

We multiply this by the volume and obtain: $E_{\text{tot}} = \left[\left(\frac{3}{\pi}\right)^{2/3} \frac{1}{5\cdot 2^{37/6}} \frac{h^2}{m_n r_n^5} - \frac{\sqrt[3]{9\pi}Gm_n^{4/3}}{20\sqrt[3]{4}r_n^4} M^{2/3}\right] \frac{4\sqrt{2}Mr_n^3}{m_n}$

O.M.G.: $E_{\text{tot}} = \left(\frac{3}{\pi}\right)^{2/3} \frac{1}{5\cdot 2^{37/6}} \frac{h^2}{m_n r_n^5} \frac{4\sqrt{2}Mr_n^3}{m_n} - \frac{\sqrt[3]{9\pi}Gm_n^{4/3}}{20\sqrt[3]{4}r_n^4} \frac{4\sqrt{2}Mr_n^3}{m_n} M^{2/3}$

i.e.: $E_{\text{tot}} = \left(\frac{3}{\pi}\right)^{2/3} \frac{4\sqrt{2}}{5\cdot 2^{37/6}} \frac{h^2}{m_n^2 r_n^2} M - \frac{4\sqrt{2}\sqrt[3]{9\pi}Gm_n^{1/3}}{20\sqrt[3]{4}r_n} M^{5/3}$

which is: $E_{\text{tot}} = \left(\frac{3}{\pi}\right)^{2/3} \frac{2^{5/2}}{5\cdot 2^{37/6}} \frac{h^2}{m_n^2 r_n^2} M - \frac{2^{5/2}\cdot\sqrt[3]{9\pi}Gm_n^{1/3}}{5\cdot 2^{2/3}r_n} M^{5/3}$

yielding: $E_{\text{tot}} = \left(\frac{3}{\pi}\right)^{2/3} \frac{1}{5\cdot 2^{11/3}} \frac{h^2}{m_n^2 r_n^2} M - \frac{2^{11/6}\cdot\sqrt[3]{9\pi}Gm_n^{1/3}}{5r_n} M^{5/3}$

so we get: $E_{\text{tot}} = \left(\frac{3}{\pi}\right)^{2/3} \frac{1}{40\cdot\sqrt[3]{4}} \frac{h^2}{m_n^2 r_n^2} M - \frac{2^{11/6}\cdot\sqrt[3]{9\pi}Gm_n^{1/3}}{5r_n} M^{5/3}$

Per neutron, this is: $E_{\text{tot}/n} = \left(\frac{3}{\pi}\right)^{2/3} \frac{1}{40\cdot\sqrt[3]{4}} \frac{h^2}{m_n^2 r_n^2} \frac{M}{\boldsymbol{N}} - \frac{2^{11/6}\cdot\sqrt[3]{9\pi}Gm_n^{1/3}}{5r_n} \frac{M^{5/3}}{\boldsymbol{N}}$

which equals: $E_{\text{tot}/n} = \left(\frac{3}{\pi}\right)^{2/3} \frac{1}{40\cdot\sqrt[3]{4}} \frac{h^2}{m_n^2 r_n^2} \frac{M}{\boldsymbol{M/m_n}} - \frac{2^{11/6}\cdot\sqrt[3]{9\pi}Gm_n^{\boldsymbol{1/3}}}{5r_n} \frac{M^{5/3}}{\boldsymbol{M/m_n}}$

hence: $\boldsymbol{E_{\text{tot}/n} = \left(\frac{3}{\pi}\right)^{2/3} \frac{1}{40\cdot\sqrt[3]{4}} \frac{h^2}{m_n r_n^2} - \frac{2^{11/6}\cdot\sqrt[3]{9\pi}Gm_n^{4/3}}{5r_n} M^{2/3}}$

For a stable ball, this must (by absolute value) exceed the neutron decay energy,

so the stability tipping point is: $\boldsymbol{E_\beta + \left(\frac{3}{\pi}\right)^{2/3} \frac{1}{8\cdot\sqrt[3]{4}} \frac{h^2}{m_n r_n^2} - \frac{2^{11/6}\cdot\sqrt[3]{9\pi}Gm_n^{4/3}}{5r_n} M^{2/3} = 0}$

from which we obtain: $M^{2/3} = \left(E_\beta + \left(\frac{3}{\pi}\right)^{2/3} \frac{1}{8\cdot\sqrt[3]{4}} \frac{h^2}{m_n r_n^2}\right) \frac{5r_n}{2^{11/6}\cdot\sqrt[3]{9\pi}Gm_n^{4/3}}$

hence: $$M^2 = \left(E_\beta + \left(\frac{3}{\pi}\right)^{2/3} \frac{1}{8\cdot\sqrt[3]{4}} \frac{h^2}{m_n r_n^2}\right)^3 \frac{125 r_n^3}{2^{11/2}\cdot 9\pi G^3 m_n^4}$$

which is: $$M^2 = \left(E_\beta + \left(\frac{3}{\pi}\right)^{2/3} \frac{1}{8\cdot\sqrt[3]{4}} \frac{h^2}{m_n r_n^2}\right)^3 \frac{125 r_n^3}{9216\sqrt{2}\pi G^3 m_n^4}$$

so the tipping mass would be: $$\boldsymbol{M = \sqrt{\left(E_\beta + \left(\frac{3}{\pi}\right)^{2/3} \frac{1}{8\cdot\sqrt[3]{4}} \frac{h^2}{m_n r_n^2}\right)^3 \frac{125 r_n^3}{9216\sqrt{2}\pi G^3 m_n^4}}}$$

Its value is: $\approx \mathbf{1.337 \times 10^{29}}$ **kg** $\approx \mathbf{6.726 \times 10^{-2}} \boldsymbol{M_\odot}$

Rougly one tenth of a solar mass.
This *would* be the minimal mass required for a stable ball of non-relativistic neutronium, taking degeneracy pressure into account.

Its radius would roughly be: $\approx \mathbf{3.998}$ **km**

This is the same as what Helstrom[6] gives ($\boldsymbol{R_{\mathrm{NR}}} \approx$ **4 km**) just below eqn. 3.8 in the last paragrah of §3.1, so it seems we can safely say the above derivation is not just some gobbledygook, although it really differs from what he does; it is way more simplistic. However, he seems to find R_{NR} using $M_{\mathrm{UR}} \approx 2M_\odot$, which, to my mind, cannot be correct. $2M_\odot$ of close-packed neutronium has a radius of $\approx$ 12.4 km.

Please note: my earlier remark:

> *Such light neutron stars have a.f.a.i.k. never been observed, so either the circumstances for forming them are in practice never met, or maybe they cannot even exist at all.*

still stands!

By the way, have you already recognised the
aforementioned pingpœunium cannot ever exist?
It would most probably completely disintegrate in a really short time!
VERY dangerous!
Remember its mass, expressed as a no. of cars?
The Hiroshima bomb contained merely 64 kg of uranium,
of which probably *less than one gram* actually fissured.
And, for both gravitational and degeneracy pressure,
the average throughout the entire ball was used,
so the above result remains merely an indicative estimate.

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*The **Blue Marble** is a quite massive stable ball (& Archimedes promised to lift it, given a proper lever and fulcrum) & more than half of its mass consists of neutrons, but it can by far not compete with close-packed neutronium.*
*The latter **would** (with $M = M_\oplus \approx 6 \times 10^{24}$ kg) have a radius of merely 142 metres ($R_\oplus \approx 6371$ km).*
This also demonstrates the emptyness of atoms (and their intermediate space, of course)
*& **YOUR** 🫵 head is even emptier...* 😉

