Gravitational potential energy of homogeneous massive sphere:
potential energy of shell = G × inner mass × shell mass / shell radius;
total potential energy = integral from nought to R:

$$U = -G\int_0^R \frac{M_{(<r)}\cdot dM_r}{r} = -G\int_0^R \frac{\left(\rho\cdot\frac{4}{3}\pi r^3\right)(\rho\cdot 4\pi r^2 dr)}{r} = \frac{-16}{3}\pi^2 G\rho^2 \int_0^R r^4 dr$$

$$U = \frac{-16}{3}\pi^2 G\rho^2 \left.\frac{r^5}{5}\right]_0^R = \frac{-16\pi^2 G\rho^2 R^5}{15}$$

$$\rho = \frac{3M}{4\pi R^3} \therefore \rho^2 = \frac{9M^2}{16\pi^2 R^6}$$

$$U = \frac{-16\pi^2 G\frac{9M^2}{16\pi^2 R^6}R^5}{15} = \frac{-3GM^2}{5R}$$

This matches: https://scienceworld.wolfram.com/physics/SphereGravitationalPotentialEnergy.html

Mean density thereof = mean gravitational pressure:

$$\bar{P}_\mathrm{g} = \frac{-3U}{4\pi R^3} = \frac{-16\pi^2\rho^2 GR^5}{15\cdot 4\pi R^3} = \frac{-4\pi G\rho^2 R^2}{15}$$

$$\bar{P}_\mathrm{g} = \frac{-3U}{4\pi R^3} = \frac{-9GM^2}{20\pi R^4}$$

Negative ⇛ compressive pressure (after all, gravitation is attractive, isn't it?).

Hubble constant, *age of universe*, *Hubble distance*:

$H = \frac{1}{t_\mathrm{H}} \approx 71$ km/s/Mpc, $D_\mathrm{H} = ct_\mathrm{H} = \frac{c}{H}$; $\{D_\mathrm{H}, t_\mathrm{H}\} \approx 13.77 \times 10^9$ (light) years

https://henk-reints.nl/astro/HR-Geometry-of-universe-slideshow.pdf:

Cosmos is a 3-sphere ($D_\mathrm{H} = \pi R_{3S}$ = *antipodal distance* = *Hubble distance*):

Mass of universe: $M_\mathrm{U} = \frac{c^3}{2GH} \approx 8.77 \times 10^{52}$ kg

therefore: $G = \frac{c^3}{2HM_\mathrm{U}} = \frac{c^3 t_\mathrm{H}}{2M_\mathrm{U}} = \frac{c^2 D_\mathrm{H}}{2M_\mathrm{U}} \approx 6.67430 \times 10^{-11}$ m³/kg/s²

cosmic volume: $V_\mathrm{U} = \frac{2D_\mathrm{H}^3}{\pi} = \frac{2c^3}{\pi H^3} \approx 1.407 \times 10^{78}$ m³ ≈ 1662 Gly³

mean density: $\bar{\rho}_\mathrm{U} = \frac{M_\mathrm{U} = \frac{c^3}{2GH}}{V_\mathrm{U} = \frac{2c^3}{\pi H^3}} = \frac{\pi H^2}{4G} = \frac{\pi\left(\frac{c}{D_\mathrm{H}}\right)^2}{4\frac{c^2 D_\mathrm{H}}{2M_\mathrm{U}}} = \frac{\pi M_\mathrm{U}}{2D_\mathrm{H}^3} \approx 6.23 \times 10^{-26}$ kg/m³

3-spherical geometry:

Volume of a sphere[1]: $V_{3S}(r) = \pi R_{3S}^3\left(\frac{2r}{R_{3S}} - \sin\frac{2r}{R_{3S}}\right) = \frac{D_\mathrm{H}^3}{\pi^2}\left(\frac{2\pi r}{D_\mathrm{H}} - \sin\frac{2\pi r}{D_\mathrm{H}}\right)$

surface area thereof: $A_{3S}(r) = \frac{dV_{3S}(r)}{dr} = \frac{D_\mathrm{H}^3}{\pi^2}\left(\frac{2\pi}{D_\mathrm{H}} - \frac{2\pi}{D_\mathrm{H}}\cos\frac{2\pi r}{D_\mathrm{H}}\right)$

$$= \frac{2D_\mathrm{H}^2}{\pi}\left(1 - \cos\frac{2\pi r}{D_\mathrm{H}}\right) = \frac{4D_\mathrm{H}^2}{\pi}\sin^2\frac{\pi r}{D_\mathrm{H}}$$

$$U_\mathrm{U} = -\int_0^{D_\mathrm{H}} \frac{G\cdot M_{(<r)}\cdot dM_r}{r} = -G\int_0^{D_\mathrm{H}} \frac{\bar{\rho}_\mathrm{U}V_{3S}(r)\cdot\bar{\rho}_\mathrm{U}A_{3S}(r)dr}{r} = -G\bar{\rho}_\mathrm{U}^2\int_0^{D_\mathrm{H}} \frac{V_{3S}(r)A_{3S}(r)}{r}dr$$

numerator within integral:

$$V_{3S}(r)A_{3S}(r) = \frac{D_\mathrm{H}^3}{\pi^2}\left(\frac{2\pi r}{D_\mathrm{H}} - \sin\frac{2\pi r}{D_\mathrm{H}}\right)\frac{2D_\mathrm{H}^2}{\pi}\left(1 - \cos\frac{2\pi r}{D_\mathrm{H}}\right)$$

$$= \frac{2D_\mathrm{H}^5}{\pi^3}\left(\frac{2\pi r}{D_\mathrm{H}} - \sin\frac{2\pi r}{D_\mathrm{H}}\right)\left(1 - \cos\frac{2\pi r}{D_\mathrm{H}}\right)$$

[1] E.g. a *sphere of latitude*, cf. a *circle of latitude* on a normal sphere (i.e. a 2-sphere).

$$= \frac{2D_H^5}{\pi^3}\left(\frac{2\pi r}{D_H} - \sin\frac{2\pi r}{D_H} - \frac{2\pi r}{D_H}\cos\frac{2\pi r}{D_H} + \sin\frac{2\pi r}{D_H}\cos\frac{2\pi r}{D_H}\right)$$

$$x \coloneqq \frac{2\pi r}{D_H} \therefore r = \frac{D_H}{2\pi}x \therefore dr = \frac{D_H}{2\pi}dx$$

$$(r = D_H) \Rightarrow x = 2\pi$$

$$\Rightarrow \frac{2D_H^5}{\pi^3}(x - \sin x - x\cos x + \sin x\cos x)$$

$$U_U = -G\bar{\rho}_U^2 \int_0^{D_H} \frac{V_{3S}(r)A_{3S}(r)dr}{r} = -G\bar{\rho}_U^2 \frac{2D_H^5}{\pi^3}\int_0^{2\pi} \frac{(x - \sin x - x\cos x + \sin x\cos x)}{\frac{D_H}{2\pi}x}\frac{D_H}{2\pi}dx$$

$$= -G\bar{\rho}_U^2 \frac{2D_H^5}{\pi^3}\int_0^{2\pi} \frac{(x - \sin x - x\cos x + \sin x\cos x)}{x}dx$$

$$\int_0^{2\pi} \frac{(x - \sin x - x\cos x + \sin x\cos x)}{x}dx = \int_0^{2\pi}\left(1 - \frac{\sin x}{x} - \cos x + \frac{\sin x\cos x}{x}\right)dx$$

$$= \int_0^{2\pi}\left(1 - \frac{\sin x}{x} - \cos x + \frac{\sin 2x}{2x}\right)dx = \left(x - \mathrm{Si}(x) - \sin(x) + \frac{\mathrm{Si}(2x)}{2}\right)\Big|_0^{2\pi}$$

$$= \mathbf{2\pi - Si(2\pi) + \frac{Si(4\pi)}{2}}$$

$\mathrm{Si}(x) = \int_0^x \frac{\sin t}{t}dt =$ ***sine integral*** *(no further analytical solution exists)*

$$U_U = -G\bar{\rho}_U^2 \frac{2D_H^5}{\pi^3}\left(2\pi - \mathrm{Si}(2\pi) + \frac{\mathrm{Si}(4\pi)}{2}\right) = -G\bar{\rho}_U^2 \frac{8D_H^5}{\pi^2}\left(\frac{1}{2} - \frac{\mathrm{Si}(2\pi)}{4\pi} + \frac{\mathrm{Si}(4\pi)}{8\pi}\right)$$

I hate ugly things... 🤮

$$\boldsymbol{K_0} \coloneqq \frac{1}{2} - \frac{\mathrm{Si}(2\pi)}{4\pi} + \frac{\mathrm{Si}(4\pi)}{8\pi} \approx 0.446\,518\,292\,037\,9218$$

$$\boldsymbol{K_1} \coloneqq 1 - \boldsymbol{K_0} \approx 0.553\,481\,707\,962\,0782$$

$$U_U = -\boldsymbol{K_0}G\bar{\rho}_U^2\frac{8D_H^5}{\pi^2} = -\boldsymbol{K_0}\frac{c^2 D_H}{2M_U}\frac{\pi^2 M_U^2}{4D_H^6}\frac{8D_H^5}{\pi^2} = -\boldsymbol{K_0 M_U c^2}$$

$$\boldsymbol{U_U = -K_0 M_U c^2 \approx -3.52 \times 10^{69}\ \mathrm{J}}$$

Does not change as time is progressing!
I.e. *cosmic expansion* requires zero energy!

The latter is in agreement with p.102 (as of 2025-07-14) of:
https://henk-reints.nl/astro/HR-Geometry-of-universe-slideshow.pdf
where I conclude the very same, based on an entirely other derivation.

$$E_U \coloneqq M_U c^2 \approx 7.88 \times 10^{69}\ \mathrm{J},\quad \boldsymbol{\bar{P}_U} \coloneqq \frac{E_U}{V_U} = \bar{\rho}_U c^2 = \frac{\pi M_U c^2}{2D_H^3} \approx 5.6 \times 10^{-9}\ \mathrm{Pa}$$

$$U_U = -\boldsymbol{K_0}E_U\,,\quad \boldsymbol{\bar{P}_{g,3S}} \coloneqq \frac{U_U}{V_U} = \frac{-\boldsymbol{K_0}E_U}{V_U} = -\boldsymbol{K_0\bar{P}_U} \approx -2.5 \times 10^{-9}\ \mathrm{Pa}$$

If we consider the mean cosmic energy density $\boldsymbol{\bar{P}_U}$ an expansive pressure,

then: $\boldsymbol{P_{tot}} \coloneqq \boldsymbol{\bar{P}_U} + \boldsymbol{\bar{P}_{g,3S}} = (1 - \boldsymbol{K_0})\boldsymbol{\bar{P}_U} = \boldsymbol{K_1\bar{P}_U}$

Net expansive cosmic pressure =

$$\boldsymbol{P_{tot}} = \boldsymbol{K_1\bar{P}_U} = \boldsymbol{K_1}\bar{\rho}_U c^2 \approx 0.5535\frac{\pi M_U c^2}{2D_H^3} \approx \boldsymbol{3.1 \times 10^{-9}\ \mathrm{Pa} > 0}$$

Gravitation is too weak to prevent cosmic expansion.

With: $D_H = \frac{c}{H}$: $\quad P_{tot} = \boldsymbol{K_1\bar{P}_U} = \boldsymbol{K_1}\frac{\pi M_U}{2c}H^3 \therefore \boldsymbol{\bar{P}_U} = \frac{\pi M_U}{2c}\boldsymbol{H^3} \therefore \boldsymbol{H} = \sqrt[3]{\frac{2c}{\pi M_U}\bar{P}_U}$

However:

Did you ever ponder the r^2 in Newton's *law of gravitation*,
or have you always haphazardly taken it for granted?

Wouldn't next be plausible?

Would *gravitational rays* exist, they'd have a *flux* $\propto \frac{1}{A(r)}$

Euclidian: $A(r) = 4\pi r^2 \therefore \frac{1}{r^2} = \frac{4\pi}{A(r)}$ 

Generalised: in "any" geometry: $F_g^* = \frac{4\pi GMm}{A(r)}$

I.e. in 3S-geometry: $F^*_{g,3S}(r) = \frac{4\pi GMm}{\frac{4D_H^2}{\pi}\sin^2\frac{\pi r}{D_H}} = \frac{\pi^2\frac{c^2D_H}{2M_U}Mm}{D_H^2\sin^2\frac{\pi r}{D_H}} = \frac{\pi^2c^2Mm}{2M_UD_H}\frac{1}{\sin^2\frac{\pi r}{D_H}}$

Gravitational potential energy w.r.t. ~~infinity~~ D_H:

$$U^*_{3S}(r) = \int_{D_H}^{r} F^*_{g,3S}(\tilde{r})d\tilde{r} = \frac{\pi^2c^2Mm}{2M_UD_H}\int_{D_H}^{r}\frac{1}{\sin^2\frac{\pi\tilde{r}}{D_H}}d\tilde{r}$$

$$x = \frac{\tilde{r}}{D_H} \therefore \tilde{r} = D_H x \therefore d\tilde{r} = D_H dx$$

$$(\tilde{r} = r) \Rightarrow x = \frac{r}{D_H} =: \rho$$

$$(\tilde{r} = D_H) \Rightarrow x = 1$$

$$U^*_{3S}(r) = \frac{\pi^2c^2Mm}{2M_UD_H}\int_1^{\rho}\frac{1}{\sin^2\pi x}D_H dx = \frac{\pi^2c^2Mm}{2M_U}\int_1^{\rho}\frac{1}{\sin^2\pi x}dx$$

$$\int_1^{\rho}\frac{1}{\sin^2\pi x}dx = \frac{-\cot(\pi x)}{\pi}\bigg|_1^{\rho}$$

$$= \frac{-\cot(\pi\rho)}{\pi} - \frac{-\cot(\pi)}{\pi}$$

Houston, we have a problem! 😞

Would it have succeeded, we should now essentially redo
the above derivation, but with the new potential energy equation.

Alas... 😞

But let's rewrite Newtonian gravitation as: $F_g = G\cdot\frac{m_1}{r}\cdot\frac{m_2}{r}$

& define ***gravitational influence*** as: $Y := \frac{m}{r}$

yielding: $F_g = G\cdot Y_1\cdot Y_2$ Ain'tn't this not unugly?

This would be were r^2 *comes from. One* r *for each mass.*

I think I can safely ditch this failed *gravitational flux theory*
& stick to what I found above.

Yours severely,
Henkus Tancus Sapiens
Henk de denktank.