

In <https://www.lehman.edu/faculty/anchordoqui/chapter25.pdf> we find:

reduced mass: $\frac{1}{\mu} = \frac{1}{m_1} + \frac{1}{m_2} \therefore \mu = \frac{m_1 m_2}{m_1 + m_2} \quad \therefore m_1 m_2 = \mu(m_1 + m_2) =: M\mu$

semilatus rectum: $r_0 = \frac{L^2}{\mu G m_1 m_2} = \frac{L^2}{GM\mu^2}$

energy: $E = \frac{L^2}{2\mu r^2} - \frac{Gm_1 m_2}{r} = \frac{L^2}{2\mu r^2} - \frac{GM\mu}{r}$

note: $L = \mu v r \quad \therefore \frac{L^2}{2\mu r^2} = \frac{1}{2} \mu v^2, \quad E = E_{\text{kin}} + E_{\text{pot}}$

$E < 0 \rightarrow$ elliptical orbit, $E = 0 \rightarrow$ parabolic orbit, $E > 0 \rightarrow$ hyperbolic orbit

excentricity: $e = \sqrt{1 + \frac{2EL^2}{\mu(Gm_1 m_2)^2}} = \sqrt{1 + \frac{2EL^2}{\mu(GM)^2 \mu^2}}$

orbit equation: $r = \frac{r_0}{1 - e \cos \theta}$

Sem. rect. of ellipse: $r_0 = a(1 - e^2) \quad \therefore a = \frac{L^2}{GM\mu^2(1 - e^2)}$

With: $\mathcal{L} := \frac{L}{\mu}$ (dimension: surface area per time)

and: $\mathcal{E} := \frac{E}{\mu}$ (dimension: squared velocity)

as well as: $\mathcal{M} := GM$ (dimension: volume per squared time)

we obtain: $e = \sqrt{1 + \frac{2\mathcal{E}\mathcal{L}^2}{\mathcal{M}^2}} \quad \therefore 1 - e^2 = \frac{-2\mathcal{E}\mathcal{L}^2}{\mathcal{M}^2}$

and: $r = \frac{\mathcal{L}^2}{\mathcal{M}(1 - e \cos \theta)}$

as well as: $a = \frac{\mathcal{L}^2}{\mathcal{M}(1 - e^2)} = \frac{\mathcal{L}^2}{\mathcal{M}} \cdot \frac{\mathcal{M}^2}{-2\mathcal{E}\mathcal{L}^2} \therefore$

$$a = \frac{\mathcal{M}}{-2\mathcal{E}}$$

We also have: $b = a\sqrt{(1 - e^2)} = \left(\frac{-\mathcal{M}}{2\mathcal{E}} \sqrt{\frac{-2\mathcal{E}\mathcal{L}^2}{\mathcal{M}^2}} \right) = \frac{\mathcal{L}}{-2\mathcal{E}} \sqrt{-2\mathcal{E}}$

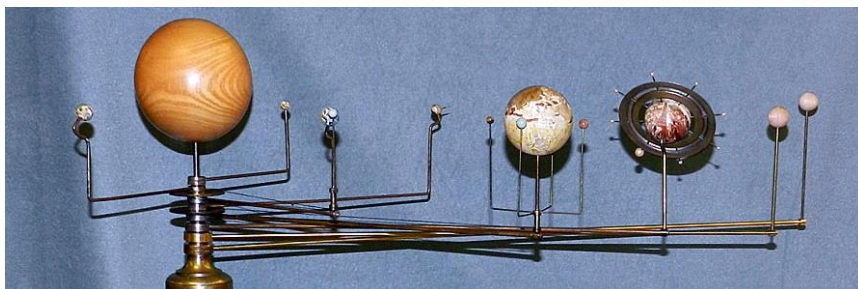
$$b = \frac{\mathcal{L}}{\sqrt{-2\mathcal{E}}}$$

"roundness": $\frac{b}{a} = \sqrt{(1 - e^2)} = \sqrt{\frac{-2\mathcal{E}\mathcal{L}^2}{\mathcal{M}^2}} \quad \frac{b}{a} = \frac{\mathcal{L}\sqrt{-2\mathcal{E}}}{\mathcal{M}}$

We define: $v := \sqrt{-2\mathcal{E}} \quad \therefore 2\mathcal{E} = -v^2 \quad a = \frac{\mathcal{M}}{v^2}, \quad b = \frac{\mathcal{L}}{v}$

(not the orbital velocity, but sort of; v decreases as r increases)

surface area: $A = \pi ab = \pi \cdot \frac{\mathcal{M}}{v^2} \cdot \frac{\mathcal{L}}{v} \quad A = \frac{\pi \mathcal{M} \mathcal{L}}{v^3}$



https://www2.humboldt.edu/scimus/MedInst_rap/Orrery/Orrerya_12.jpg